Abstract

Essays in Accounting Theory: Corporate Earnings Management in a Dynamic Setting and Public Disclosure in the Financial Services Industry

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This dissertation consists of three essays on the interactions between economic fundamentals and accounting information in three different settings: an infinite-horizon financial reporting problem, a coordination game with trading in the secondary market, and a bank which provides risk sharing among demand depositors. In the first essay, I propose a dynamic model of corporate earnings management in which investors have different expectations schemes. I find that while earnings management may exist when investors have rational expectations or misspecified Bayesian beliefs, it disappears in the long run of an adaptive learning process. The model also offers ample predictions on the time-series properties of asset prices and return predictabilities. The second essay studies the role of public disclosure by a distressed firm whose creditors engage in a coordination game with trading. I find that conditioned on the private information environment and equilibrium selection, better public disclosure could lead to higher probability of creditor run and lower expected welfare. The third essay introduces loan loss provision to a bank that transforms illiquid assets into demand deposits, and shows that the contingency in demand-deposit contracts induced by loan loss provision reduces risk sharing.

Essays in Accounting Theory:
Corporate Earnings Management in a Dynamic Setting and Public Disclosure in the Financial Services Industry

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Chapter 1

Introduction

This dissertation studies the interactions between economic fundamentals and accounting information in three different settings: (i) a dynamic setting of earnings management, (ii) a coordination game played by creditors who also trade assets of the firm they finance, and (iii) a financial intermediation setting in which loan loss provision changes the structure of the demand deposit contracts. The three essays extend the scope of accounting theories, and offer empirical predictions or policy implications.

In the first essay "A Dynamic Model of Investor Expectations and Financial Reporting," I examine the implications of investor expectations for earnings management and asset prices. I propose an infinite-horizon stochastic model of financial reporting, in which a firm manager opportunistically shifts earnings in order to manipulate investor expectations, subject to a constraint on accumulated accruals. Three investor expectations schemes with different levels of rationality are considered: rational expectations, adaptive learning, and misspecified Bayesian learning. When investors have rational expectations, a truthful reporting equilibrium may arise when real earnings are highly persistent. Under adaptive learning, earnings management only exists in the early stage when there is a discrepancy between investor beliefs and the actual process of reported earnings. Under regime-shifting beliefs, the manager always engages in earnings management as the perceived earnings process never converges to the true process. I also find an inverted-U-shaped relationship between the severity of investor sentiment and earnings management. By simulating the relationship between accounting information and future stock returns, I find that the regime-shifting scheme is better able to approximate several empirical regularities.

In the second essay "Information Aggregation and Multiplicity in Creditor Runs: The Role of Public Disclosure," I study the role of public disclosure in a coordination game, in which creditors trade a financial asset of the firm they finance, before deciding whether to rollover lending to the firm. Creditors base their trading and lending decisions on three sources of information: private information, public disclosure, and asset prices. I show that public disclosure plays a critical role in equilibrium outcome of corporate default, through its interaction with private information. Specifically, when public disclosure is sufficiently noisy, unique equilibrium only obtains for intermediate values of the precision of private information; when public disclosure is sufficiently precise, multiple equilibria always emerge. The impact of public disclosure on ex ante run probability and creditor welfare is conditioned on the precision of private information, and (when multiple equilibria exist) which equilibrium is played. I also show that the results hold for at least two categories of financial assets, equity shares and credit derivatives.

The third essay "Demand Deposits, Loan Loss Provision, and Risk Sharing" develops a framework for studying the role of loan loss provision in a bank that transforms illiquid assets into demand deposits, and shows that loan loss provision introduces contingency to demand-deposit contracts to the detriment of risk sharing. Building on a variant of the Diamond-Dybvig (1983) model, I assume that a bank makes loan loss provision based on an interim signal that indicates a change in the expectation of loan payoff, even though such signal may be uninformative. I solve for the optimal demand-deposit contract which is contingent on loan loss provision, and show that such contract induces underinvestment in the risky loan and leads to lower expected utility for depositors, relative to cases where there is either no interim information or perfect interim information. I conclude that in the presence of liquidity risk, uninformative loan loss provision may generate undesirable contingency that leads to suboptimal risk sharing. This finding provides theoretical justification for accountants' adherence to the informational role of loan loss provision, in spite of regulators' campaign for counter-cyclical loan loss provision.

Chapter 2

A Dynamic Model of Investor Expectations and Financial Reporting

2.1 Introduction

Investor expectations play a central role in explaining capital market-driven earnings management. Investors do not usually have complete knowledge about the environment of financial reporting, and have to form expectations in the presence of structural and parameter uncertainties, or using potentially misspecified models. Aware of this, firm managers can influence stock prices by opportunistically shifting earnings over time. The interplay between investor expectations and managerial discretion is important for understanding the joint determination of earnings and price; however, this has not yet been formulated in a dynamic setting.

In this paper, I propose a dynamic stochastic model of financial reporting that integrates investor expectations, managerial opportunism, and accruals constraints. The real earnings of a firm are generated by a stochastic process, but the firm manager may shift earnings over time by creating accounting accruals, while she is required to keep the level of accumulated accruals within a certain range. For each period, the manager's reporting decision maximizes her expected utility derived from stock prices, in the presence of real earnings uncertainty. Investors, who do not observe real earnings, form expectations of future earnings based on reported earnings: They may rationally anticipates manager's reporting strategy (rational expectations); alternatively, they may act as econometricians and fit a reduced-form forecasting relationship recursively (adaptive learning); or they may perceive reported earnings as generated by one of two different regimes, and update their beliefs about the prevailing regime in a Bayesian fashion (regime-shifting beliefs). These scenarios are representative of a wide range of investor expectations schemes, as proverbially known to practitioners or assumed in academic studies. They vary not only in terms of investor rationality, but also in the knowledge base of investors' decision making. Because investor rationality and the completeness of knowledge cannot be easily parameterized, an understanding of their roles in earnings management requires a collective study of various expectations schemes.

I begin with a stationary rational expectations equilibrium in which the probability distribution of state variables is invariant over time, and show that whether the manager truthfully reports earnings in equilibrium depends on the underlying real earnings process. I then depart from the "full knowledge/rational expectations" benchmark in two directions. One direction is adaptive learning, in which investors confront structural uncertainty with respect to the manager's reporting problem, but recursively fit a statistical model of reported earnings. By not granting investors a greater degree of knowledge than an econometrician who strives to estimate model parameters from historical data, this assumption seems more plausible than rational expectations. The other direction is

Bayesian learning with regime-shifting beliefs, in which investors believe in misspecified models of the earnings process. This expectations scheme is consistent with experimental evidence on investors' forecasting behavior, and is especially relevant for a world populated by short-term investors whose horizons are not long enough to learn the true structure of the reporting problem.

For each expectations scheme, I formulate the manager's reporting strategy as the solution to a dynamic programming problem. I find that when investors do not have rational expectations, the manager's misreporting decisions are characterized by threshold rules conditioned on investors' beliefs. For instance, in the case of regime-shifting beliefs, the manager deflates earnings if investors perceive the earnings process as mean-reverting with a high enough probability; the threshold probability is higher when accumulated accruals are closer to hitting the constraint. Apparently, the manager weighs the capital market benefit (cost) of earnings management against the cost (benefit) associated with depleting (increasing) "cookie jar" reserves, which affects her future leeway in managing earnings.

Most importantly, the existence of earnings management and the path of investor beliefs depend crucially on the expectations scheme. In a stationary rational expectations equilibrium, earnings management is more likely to be an equilibrium when real earnings are less persistent. In the case of adaptive learning, there is earnings management in early periods when the investors' perceived law of motion still deviates from the actual law of motion;[1] as investors learn the actual law of motion over time, the manager ceases to manage earnings. Under Bayesian learning with regime-shifting beliefs, the perceived law of motion never converges to the actual one, and earnings management is always prevalent.

[1]Following the conventions of models with learning, the true model is referred to as the actual law of motion, and the model used by agents inside the model to form expectations is referred to as the perceived law of motion.

This model generates many testable predictions, and replicates several forms of return regularity. Under regime-shifting beliefs, the prevalence of earnings management exhibits an inverted-U-shaped relationship with respect to regime heterogeneity as perceived by investors. Value relevance, mean return, and Sharpe ratio also vary with the severity of model misspecification. By simulating a large number of idiosyncratic firms and forming portfolios based on earnings or accruals, the model can be used to simulate accounting-based return predictabilities. Under adaptive learning, there is overreaction but no underreaction to earnings news; under regime-shifting beliefs, both underreaction and overreaction obtain. The accruals anomaly is generated by both adaptive learning and regime-shifting beliefs, with a larger magnitude under adaptive learning. I also examine the implications of managerial opportunism for accounting-based return anomalies. Under regime-shifting beliefs, short-term underreaction to managed earnings is less severe (more severe) than it is to unmanaged earnings when real earnings are mean-reverting (persisting).

Prior literature. This paper relates to three strands of research: (i) studies on the asset pricing implications of various investor learning schemes, (ii) the earnings management literature, and (iii) studies on corporate decisions in the presence of capital market inefficiencies.

A growing literature investigates the implications of various investor learning schemes for asset price dynamics.[2] Some studies adopt adaptive learning schemes such as least-square learning and maximum-likelihood learning (e.g., Timmermann 1993, 1996; Lewellen and Shanken 2002; Guidolin and Timmermann 2007; Evans and Honkapohja 2001; Hansen and Sargent 2007). Others examine Bayesian learning based on misspecified beliefs, such as regime-shifting beliefs (e.g., Barberis, Shleifer, and Vishny 1998), certain statistical dis-

[2]The concept of "learning" as used in this literature is different from the game-theoretic learning (e.g., Fudenberg and Levine 1998; Young 2004).

tributions (e.g., Guidolin and Timmermann 2007), simple forecasting models (e.g., Hong, Stein, and Yu 2007; Branch and Evans 2010), and behavioral heuristics (e.g., Brav and Heaton 2002). There are also asset pricing models of rational expectations with structural uncertainties (e.g., Veronesi 1999; Brav and Heaton 2002; Guidolin and Timmermann 2007). These studies invariably assume that earnings/dividends process are *exogenously* presented, and do not model corporate managers' decisions.[3] For example, Barberis et al. (1998) study the asset pricing implications of regime-shifting beliefs when earnings follow an exogenous random process. In this paper however, earnings are endogenous in the sense that they are opportunistically reported by a manager. Besides asset pricing literature, studies on monetary policies have also examined the implications of different public expectations (e.g., Sargent and Wallace 1975).

Second, this paper contributes to the literature of earnings management. Canonical theories of earnings management examine various settings in which managers' incentives to manipulate earnings are driven by short-term interests. For examples, Dye (1988) studies an over-lapping generation model in which earnings management arises as current shareholders attempt to influence prospective investors' expectations; Trueman and Titman (1988) propose an explanation for income smoothing based on conflicting interests among stakeholders; Fudenberg and Tirole (1995) provide an explanation for income smoothing based on managerial labor market considerations and overweighting of current performance relative to past performance; Fischer and Verrecchia (2000) study a rational expectations model of reporting bias to manipulate the market valuation of the firm. This paper identifies another two important factors that underlie earnings manipulations:

[3]In addition to asset pricing settings, researchers have also investigated the implications of learning in the setting of analyst forecasts. For examples, Chen, Francis, and Jiang (2005) examine investors' learning about analyst predictive ability; Markov and Tamayo (2006) examine *analysts'* learning in the presence of parameter uncertainty. Similar to studies on asset pricing, both Chen et al. (2005) and Markov and Tamayo (2006) assume an exogenous earnings process.

investor beliefs about the intertemporal process of earnings, and the proximity to the boundary of a permissible range of accumulated accruals.

Several theoretical studies have taken a multi-period perspective.[4] Among these studies, Gonedes (1972) examines the reporting problem of a manager who prefers smoother earnings, without explicitly considering capital markets and investor expectations; Stein (1999) proposes a signal-jamming model in which a firm manager biases earnings but investors rationally filters the earnings management in equilibrium; Ewert and Wagenhofer (2005) study a two-period model with unwinding of earnings management in the second period; Marinovic (2011) examines earnings management when the internal control system fails. This study joins this line of research by proposing a dynamic theory of earnings management in which a manager attempts to cater to investor expectations while maintaining an accruals constraint.

Finally, financial reporting is just one of the decisions that managers routinely make. A recent literature has examined other corporate decisions in a world where investors are not fully rational. For example, Baker and Wurgler (2004) examine how varying investor sentiment explains dividend payout policies. There is also accumulating empirical evidence of opportunistic disclosure or reporting in response to investor sentiment. For examples, Bergman and Roychowdhury (2008) find that managers use disclosure policies to influence investor expectations; Rajgopal et al. (2007) find that managers inflate earnings when investors react optimistically to positive earnings surprises. This paper formalizes the notion by modeling the opportunistic financial reporting decisions in response to some form of bounded rationality that characterizes investors' decision process.

The rest of the paper is organized as follows. Section 2.2 introduces the main ingredi-

[4]There are empirical or empirically orientated studies that also address the role of economic shocks from a dynamic perspective (e.g., DeFond and Park 1997; Gerakos and Kovrijnykh 2010). There are also studies on strategic disclosure in a repeated game (e.g., Stocken 2000; Golosov, Skreta, Tsyvinski, and Wilson 2011), but these studies are orthogonal to this paper.

ents of the model. Sections 2.3, 2.4, and 2.5 study three alternative investor expectations schemes, respectively: rational expectations, adaptive learning, and regime-shifting beliefs. In each scheme, I characterize the optimal reporting strategies, investor beliefs, and price functions. Section 2.6 studies the dynamics of earnings and price, focusing on accounting-based anomalies. Section 2.7 explores potential directions of extending the model. Section 2.8 concludes.

2.2 The Model

This paper proposes a dynamic stochastic model of how a corporate manager manipulates earnings to influence investor expectations. In this section, I introduce the main ingredients of the model: earnings process, investor expectations and stock price, and the manager's optimization problem.

2.2.1 Earnings process and financial reporting

Earnings process. A firm operates in a stochastic environment, and generates real earnings that can be high (H) or low (L), i.e., $\theta_t \in \{\theta_H, \theta_L\} \equiv \Theta$, for each period t. The earnings process is characterized by a Markov process with parameter π, i.e., $\Pr(\theta_{t+1} = \theta_i|\theta_t = \theta_i) = \pi$ and $\Pr(\theta_{t+1} = \theta_i|\theta_t = \theta_j) = 1 - \pi$, where $i, j \in \{H, L\}$, $i \neq j$, and $\pi \in [0, 1]$. Real earnings are privately observed by the firm manager, who is required to make a public statement about the firm's earnings. In the earnings report, the manager may inflate or deflate earnings through accounting accruals. In each period t, the reported earnings $y_t \in \{\theta_H, \theta_L\}$ is the sum of real earnings θ_t and the accruals component of earnings, $a_t \in \{A, 0, -A\}$, where $A \equiv \theta_H - \theta_L$,

$$y_t = \theta_t + a_t \tag{2.2.1}$$

The accumulated level of accruals is given by

$$x_t = x_{t-1} + a_t \tag{2.2.2}$$

I impose a constraint on the accumulated effects of income shifting: the absolute value of accumulated accruals cannot exceed A. In other words, the accumulated accruals can only take three values, $x_t \in \{-A, 0, A\} \equiv X$. The accumulated level of accruals can be understood as net operating assets which reflect the cumulative deviation between reported income and cash value added (e.g., Hirshleifer et al. 2004).[5] The intuitive justification of this "balance sheet constraint" is that the manager's flexibility to opportunistically report earnings is constrained by the extent to which the balance sheet overstates net assets relative to a neutral application of GAAP (e.g., Barton and Simko 2002; Baber et al. 2011).

This constraint gives rise to an intertemporal link between reporting decisions in different periods. Upward earnings management reduces the manager's future flexibility in inflating earnings, while downward earnings management improves it. In this regard, this study introduces a dynamic dimension to the cost and benefit[6] of earnings management. This dimension has been absent in most one-period models (e.g., Fischer and Verrecchia 2000) and multi-period models of earnings management (e.g., Stein 1989), which invariably assume an exogenous (convex) cost associated with the magnitude of reporting bias.[7]

Investor expectations. The capital market is populated by risk neutral investors who set

[5] Caution has to be taken in making the connection between the accruals concept in the model and net operating assets. In the model, the real earnings and accounting accruals are not observable to outsiders. This is the limiting case of the real world scenario in which investors do not fully observe or understand the free cash flows and accounting accruals.

[6] Cost and benefit are from a manager's perspective, instead of investors' or societal perspectives.

[7] Another study that gives substance to the cost of earnings management is by Kedia and Philippon (2009), who argue that besides inflating earnings, low-productivity firms have to excessively hire and invest in order to pool with high productivity firms.

stock price in each period based on their expectations of next period's price and earnings. The stock price at t is given by

$$p_t = \frac{1}{1+\delta} E_t[p_{t+1} + \theta_{t+1} | y_t] \tag{2.2.3}$$

where δ is investors' discount rate.[8]

Manager's optimization problem. The manager has full knowledge about investor beliefs,[9] and maximizes her personal utility. In each period, the manager consumes a fraction α of the stock price. (It is clear that the value of α is irrelevant for the manager's reporting decisions. Therefore, I normalize α to 1 for algebraic simplicity.) This assumption has two empirical underpinnings. First, executive bonuses and equity-based incentives are usually tied to stock prices. Second, empirical and survey evidence suggests that capital market reaction is a primary concern for managers when reporting financial results (e.g., Healy and Wahlen 1999; Graham et al. 2005). The per period utility of the manager is represented by a logarithm function, i.e., $u(c) = \ln(c)$, where c is her consumption.

The manager's financial reporting decision y_t maximizes her expected utility,

$$\max_{y_t \in \{\theta_H, \theta_L\}} E_t\Big[\sum_{s=0}^{\infty} \beta^s \ln(p_{t+s})\Big] \tag{2.2.4}$$

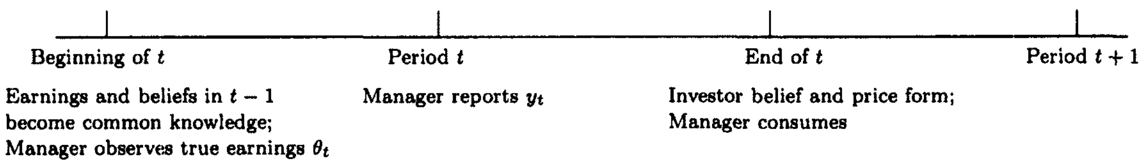
where β is the manager's discount rate (time preference), and p_t is the stock price at the end of period t. In other words, when choosing the level of reported earnings, the manager is concerned about the stream of prices from the current period onwards. Reporting decision in the current period has intertemporal implications through two channels: investor

[8]This price function can be motivated by an over-lapping generation model with risk neutral investors. The price function can also be generalized to one that compensates for variance risk, such as a mean-variance linear asset pricing model (e.g., DeLong et al. 1990).

[9]This assumption is in contrast with Amershi and Sunder (1987), who show that a rational capital market may still fail to discipline managers who hold incorrect beliefs about investors' behavior and decision rules.

expectations and the manager's future flexibility of manipulating earnings.

The timeline of the model is summarized as follows.



2.2.2 Investor expectations

The focus of this paper is how different investor expectations schemes affect the joint determination of reported earnings and asset prices. I study three expectations schemes: rational expectations, adaptive learning, and misspecified Bayesian learning.

(i) *Rational expectations.* In a rational expectations equilibrium, investors' beliefs of manager's reporting strategy is consistent with the actual reporting strategy.

(ii) *Adaptive learning.* Under adaptive learning, investors behave like econometricians in forecasting the future states of the world. In each period, investors fit a reduced-form relationship between observed data and hidden states, and use this relationship to forecast future earnings. They adjust their estimates as new data become available over time.

(iii) *Bayesian learning with regime-shifting beliefs.* Under this scheme, investors update their beliefs in a Bayesian fashion, but based on a misspecified model. Investors take reported earnings as real earnings, and perceive reported earnings as governed by one of the two regimes: a mean-reverting regime and a persisting regime.

Details about the three investor expectations schemes will be discussed in the next three sections.

2.3 Rational Expectations

Under rational expectations, investors have full knowledge about the reporting environment and model parameters, and correctly anticipate how earnings are biased for every contingency. This assumption has been maintained in theories of earnings management (e.g., Fischer and Verrecchia 2000; Ewert and Wagenhofer 2005).

In a dynamic setting, rational expectations also require that investors form beliefs of the probability distribution of the hidden states (real earnings and accumulated accruals), which in turn has to be consistent with the equilibrium distribution induced by price functions and manager's optimal decision rules. In other words, investors rationally anticipate not only manager's reporting strategy, but also the transition function and the probability distribution of state variables. In this section, I define a stationary equilibrium with rational expectations, and solve for the optimal reporting strategy, investor beliefs, and the pricing function.

2.3.1 A stationary rational expectations equilibrium

In dynamic stochastic models, it is standard to focus on equilibrium with stationary distributions (e.g., Stokey, Lucas, and Prescott 1989). Applications of the concept of stationary distributions include studies on equilibrium distributions of heterogeneous agents in macroeconomies (e.g., Huggett 1993; Aiyagari 1994), and one-sector economic growth models (e.g., Brock and Mirman 1972).[10] Studying non-stationary equilibrium would pose more technical difficulty (e.g., Hopenhayn and Prescott 1992). Following this literature, I study a particular rational expectations equilibrium (REE) in which the state variables

[10]In a similar vein, the stationarity condition is also adopted in asset pricing literature. For example, DeLong et al. (1992) consider only "steady-state" equilibria in which the unconditional distribution of price is identical over time.

have a joint distribution that is invariant over time.

In this model, the manager conditions reporting decisions on the true earnings, θ_t, and the accumulated accruals at the end of the previous period, x_{t-1}. The space of the state vector is given by $S \equiv \Theta \times X = \{\theta_H, \theta_L\} \times \{-A, 0, A\}$. Let λ be a probability measure on S. Given the discrete values of earnings and accruals, λ is simply a 6×1 vector representing the probabilities of the six possible states.[11]

Definition 2.1. A *stationary rational expectations equilibrium* is a set of decision rules $y(\theta_t, x_{t-1})$, a price function $p(y_t)$, and a probability measure on the state space $\lambda : S \to R$ such that

(i) Reporting decision is a solution to the manager's optimization problem, i.e., $y(\theta_t, x_{t-1})$ solves the following problem

$$v(\theta_t, x_{t-1}) = \max_{y_t \in \{\theta_H, \theta_L\}} \left\{ \ln\left(p(y_t)\right) + \beta E_t[v(\theta_{t+1}, x_t)] \right\} \tag{2.3.1}$$

where $v(\cdot)$ is the value function.

(ii) Investor expectations are consistent with λ and $y(\cdot)$; stock price is formed based on investor expectations.

(iii) The probability measure λ is a stationary, i.e.,

$$\lambda' = \lambda' P \tag{2.3.2}$$

where P is the 6×6 transition matrix implied by the model. An example of the stationarity condition is given by condition (2.9.6).

The discreteness of state variables allows for a reformulation of the Bellman equa-

[11] For a more general case, refer to the appendix (Section 2.9.2) for a discussion of probability measures on state spaces with (at least one) continuous state variables.

tion in terms v_{ij}, where $i \in \{H, L\}$ and $j \in \{-, 0, +\}$ correspond to $\theta_t \in \{\theta_H, \theta_L\}$ and $x_{t-1} \in \{-A, 0, A\}$, respectively. For example, v_{H-} is the value function when $\theta_t = \theta_H$ and $x_{t-1} = -A$. The same notational rules apply to y_{ij} and λ_{ij}. Let $y = [y_{H-}, y_{L-}, y_{H0}, y_{L0}, y_{H+}, y_{L+}]$, and $\lambda = [\lambda_{H-}, \lambda_{L-}, \lambda_{H0}, \lambda_{L0}, \lambda_{H+}, \lambda_{L+}]$ denote the vectors of decision rules and probability distribution, respectively.

Proposition 2.1. The Bellman equation (2.3.1) can be represented by six equations of v_{ij}, where $i \in \{H, L\}$ and $j \in \{-, 0, +\}$,

$$\begin{aligned}
v_{H-} &= \ln(p_H) + \beta[\pi v_{H-} + (1-\pi)v_{L-}] \\
v_{L-} &= \max\{\ln(p_H) + \beta[(1-\pi)v_{H0} + \pi v_{L0}], \ln(p_L) + \beta[(1-\pi)v_{H-} + \pi v_{L-}]\} \\
v_{H0} &= \max\{\ln(p_H) + \beta[\pi v_{H0} + (1-\pi)v_{L0}], \ln(p_L) + \beta[\pi v_{H-} + (1-\pi)v_{L-}]\} \\
v_{L0} &= \max\{\ln(p_H) + \beta[(1-\pi)v_{H+} + \pi v_{L+}], \ln(p_L) + \beta[(1-\pi)v_{H0} + \pi v_{L0}]\} \\
v_{H+} &= \max\{\ln(p_H) + \beta[\pi v_{H+} + (1-\pi)v_{L+}], \ln(p_L) + \beta[\pi v_{H0} + (1-\pi)v_{L0}]\} \\
v_{L+} &= \ln(p_L) + \beta[(1-\pi)v_{H+} + \pi v_{L+}]
\end{aligned} \tag{2.3.3}$$

The interpretation of the equations is straightforward. When real earnings are high (low) and accumulated accruals are negative (positive), there is no room for earnings management, and the value function is simply the utility from truthfully reporting (v_{H-} and v_{L+}); in other cases, the manager can either truthfully report earnings or manage earnings, and she will choose the strategy that maximizes her expected utility.

2.3.2 Solution

The following lemma states that in a stationary rational expectations equilibrium, the price function does not depend on the decision rules and probability distribution of the state vector.

Lemma 2.1. In a stationary rational expectations equilibrium, price function is given by

$$p(y_t) = \left[\left(I - \frac{1}{1+\delta}\Pi\right)^{-1}\bar{\theta}\right]_j \tag{2.3.4}$$

where $\Pi = [\pi, 1-\pi; 1-\pi, \pi]$, $\bar{\theta} = [\theta_H, \theta_L]'$, I is a 2×2 identity matrix, $j = 1$ if $y_t = \theta_H$, and $j = 2$ if $y_t = \theta_L$.

I adopt a "guess and verify" method to solve for stationary REE. I start with a possible reporting strategy, derive the stationary probability distribution associated with this strategy, and verify whether and under what conditions it is incentive-compatible. The results are summarized in the following proposition.

Proposition 2.2. There exist the following stationary rational expectations equilibria, in which stock prices are given by (2.3.4), decision rules $y = [y_{H-}, y_{L-}, y_{H0}, y_{L0}, y_{H+}, y_{L+}]$ and probability distribution $\lambda = [\lambda_{H-}, \lambda_{L-}, \lambda_{H0}, \lambda_{L0}, \lambda_{H+}, \lambda_{L+}]$ are given by the following,

(i) $y = [\theta_H, \theta_H, \theta_H, \theta_H, \theta_L, \theta_L]$ and $\lambda = [0, 0, \frac{\pi}{2}, \frac{1-\pi}{2}, \frac{1-\pi}{2}, \frac{\pi}{2}]$, for $\pi \in (\underline{\pi}_1, \bar{\pi}_1)$;

(ii) $y = [\theta_H, \theta_L, \theta_H, \theta_H, \theta_L, \theta_L]$ and $\lambda = [w, w, \frac{\pi}{2}(1-2w), \frac{1-\pi}{2}(1-2w), \frac{1-\pi}{2}(1-2w), \frac{\pi}{2}(1-2w)]$, where $w \in [0, \frac{1}{2}]$, for $\pi \in (\underline{\pi}_2, 1]$;

(iii) $y = [\theta_H, \theta_L, \theta_L, \theta_H, \theta_L, \theta_L]$ and $\lambda = [\frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0]$, for $\pi \in (\underline{\pi}_3, \bar{\pi}_3)$;

where $(\underline{\pi}_1, \bar{\pi}_1, \underline{\pi}_2, \underline{\pi}_3, \bar{\pi}_3)$ are constants between 0.5 and 1.

The stationary distribution λ determines which states are possible in the equilibrium. If all possible states with non-zero probabilities are characterized by truthful reporting, then the equilibrium is called a truthful reporting equilibrium. Otherwise, it is an earnings management equilibrium. Apparently, equilibrium (iii) is a truthful reporting equilibrium, while (i) and (ii) are earnings management equilibria. Therefore, both truthful reporting and earnings management are possible in stationary rational expectations equilibria. Which of the three equilibria obtains (if any equilibrium exists at all) depends on the properties of the underlying real earnings process.

The expressions for the boundary conditions on π in terms of exogenous variables are omitted for brevity. For a baseline calibration $(\theta_H, \theta_L, \beta, \delta)$=(3, 1, 0.95, 0.06), the boundaries for the conditions are given by $(\underline{\pi}_1, \bar{\pi}_1, \underline{\pi}_2, \underline{\pi}_3, \bar{\pi}_3) = (0.735, 0.880, 0.826, 0.942, 0.995)$. In fact, it can be shown numerically that the relationship $\underline{\pi}_1 \leq \underline{\pi}_2 \leq \underline{\pi}_3$ is robust to various calibrations. Therefore, truthful reporting is only possible when real earnings are highly persistent (but less persistent than a constant). In the above calibration, the condition for the truthful reporting equilibrium is $\pi \in [0.942, 0.995]$.

Note that for some values of π, there exist multiple stationary REE. For example, when $\pi = 0.85$, both the first and the second equilibria are possible. The multiplicity of equilibria in dynamic rational expectations models is standard in the literature (e.g., Evans and Honkapohja 2001; Driskill 2006), and is not the focus of this paper.[12]

In sum, both truthful reporting and earnings management are possible in stationary REE. Whether the manager truthfully reports earnings depends on the properties of the underlying earnings process.

2.4 Adaptive Learning

2.4.1 Investor beliefs under adaptive learning

Rational expectations impose restrictive epistemic assumptions: Investors must have full knowledge about the structure and parameters of the manager's financial reporting problem, and the underlying stochastic process that describes the true earnings. These assumptions are demanding for a typical (even sophisticated) investor. Sargent (1993)

[12] In this case, because price function is uniquely determined, there is no need to resort to equilibrium selection criteria such as E-stability as proposed in Evans and Honkapohja (2001) and others surveyed by Driskill (2006) to further restrict of the set of equilibrium of interest.

and Evans and Honkapohja (2001) argue that, even researchers who postulate rational expectations do not know the parameter values and must estimate them econometrically.[13] This point becomes the main motivation for Hansen and Sargent (2007):

> "Although the artificial agents within a rational expectations model trust the model, a model's author often doubts it, especially when calibrating it or after performing specification tests. There are several good reasons for wanting to extend rational expectations models to acknowledge fear of model misspecification. First, doing so accepts Muth's (1961) idea of putting econometricians and the agents being modeled on the same footing: because econometricians face specification doubts, the agents inside the model might too." (Hansen and Sargent 2007, p. 4)

In fact, rational expectations may reflect the long-run outcome of learning:

> "Muth's 'rational expectations' hypothesis, by its exclusion of a learning procedure, explicitly assumes a situation of knowledge on the part of economic agents that casts models in which it is used in mold most commonly associated with long-run equilibrium economics." (Friedman 1979, p. 25)

Therefore, it is more descriptive of the real-world belief formation process to assume that investors forecast future earnings in a similar way to econometricians. There is abundant anecdotal evidence that market participants recursively fit reduced-form forecasting models, and use the estimated relationships to project future earnings (see Brown 1993 for a review). There is also evidence that adaptive learning is the process that leads to rational expectations (e.g., Timmermann 1994).

Following this line of arguments, in this section I study the case in which investors do not know π of the underlying earnings process, but recursively estimate a statistical model of *reported* earnings (the only observable variable) in each period, and use it to form expectations about future earnings.[14] Specifically, I assume that in any period t,

[13]This statement mainly refers to the large part of the adaptive learning literature that focuses on macroeconomic subjects such as monetary policies and business cycles. For reviews of recent advances in this literature, see Evans and Honkapohja (2009; 2011).

[14]The justification for this form of adaptive learning is provided in Section 2.4.2.

investors use all past data to arrive at a maximum likelihood estimator of the probability of *earnings continuation* (defined as the event that current-period reported earnings are equal to previous-period reported earnings).[15] The estimator is denoted as $\hat{\pi}_t$,

$$\hat{\pi}_t = \frac{1}{t}\sum_{s=1}^{t} \mathbf{1}_{y_s=y_{s-1}} = \left(1-\frac{1}{t}\right)\hat{\pi}_{t-1} + \frac{1}{t}\mathbf{1}_{y_t=y_{t-1}} \tag{2.4.1}$$

where $\mathbf{1}_{y_s=y_{s-1}}$ is an indicator that takes the value 1 if $y_s = y_{s-1}$ and 0 otherwise. The idea of recursive learning is evident in the second equality: In period t, after observing one y_t, investors update their estimate of $\hat{\pi}_t$ by adjusting weights attached to historical data and the initial prior. When t is small, investors' prior plays a significant role; as t becomes large and more data come out, the role of prior and older data is diluted. The price function is given by the following lemma.

Lemma 2.2. Under adaptive learning as described by (4.1), price function is given by

$$p(y_t, \hat{\pi}_t) = \left[\left(I - \frac{1}{1+\delta}\hat{\Pi}\right)^{-1}\bar{\theta}\right]_j \tag{2.4.2}$$

where $\hat{\Pi} = [\hat{\pi}_t, 1-\hat{\pi}_t; 1-\hat{\pi}_t, \hat{\pi}_t]$, $\bar{\theta} = [\theta_H, \theta_L]'$, I is a 2×2 identity matrix, $j = 1$ if $y_t = \theta_H$, and $j = 2$ if $y_t = \theta_L$.

2.4.2 Convergence

Before proceeding to manager's optimization problem, I first explain why adaptive expectations based on *reported* earnings will converge.

Convergence of adaptive learning to rational expectations equilibrium has been analyzed in several studies, which apply Ljung's (1977) ordinary differential equation technique

[15] A conceivable modification of the model setup is to assume investors use a limited history of past data. However, the extreme case would become one with overweighting of data with greatest recency, and can be termed as "representativeness" heuristic instead of adaptive learning. As Chen et al. (2005) argue, Bayesian and non-Bayesian models alike can predict increasing weights on more recent data.

to establish convergence of linear stochastic systems with feedback. Most relevant to this study are Marcet and Sargent (1989a, 1989b). In particular, Marcet and Sargent (1989a) study the convergence of recursive least-squares learning in an asymmetric information setting where private information leads to hidden state variables from the viewpoint of less informed agents. In this study, investors are informationally disadvantaged because they do not observe two hidden state variables: real earnings and accumulated accruals. Marcet and Sargent (1989b) study "self-referential" models in which the law of motion perceived by agents influences law of motion that they actually face. This provides an abstraction of the model in this section: In each period investors estimate a perceived law of motion for reported earnings, y_t; clearly, investor expectations feed back into manager's decision making and therefore affect the actual law of motion for y_t. In other words, the manager's optimization problem induces a mapping from a perceived law of motion for y_t to the actual law of motion. With self-referentiality, Marcet and Sargent (1989b) have proved the convergence of perceived law of motion under certain conditions, which is restated in Lemma 2.3 in the language of the current setting:[16]

Lemma 2.3. (Convergence of adaptive learning in self-referential models) Under certain regularity conditions, the perceived law of motion and the actual law of motion for reported earnings y_t converge to one another.

It is easy to show that the price function will converge to one based on rational expectations.

Lemma 2.4. (Equivalence of price functions) When the perceived law of motion and the actual law of motion for reported earnings y_t coincide, the stock price formed by assuming

[16]Essentially, the regularity conditions stipulate the properties of an ordinary differential equation which involves the operator that maps the perceived law of motion into the actual law of motion. The regularity conditions are satisfied in this paper, even though no formal proof is provided due to its applied nature. Also see Marcet and Sargent (1988) for an informal summary of the results of Marcet and Sargent (1989a, 1989b).

investors price future reported earnings is the same as the stock price of a stationary rational expectations equilibrium,

$$\lim_{t\to\infty} p(y_t, \hat{\pi}_t) = p^{\text{REE}}(y_t) \tag{2.4.3}$$

The two lemmas provide for the convergence to a stationary rational expectations equilibrium of an adaptive learning scheme in which investors estimate a perceived law of motion for *reported* earnings. In fact, adaptive learning provides a selection criterion (as an asymptotic justification) for the rational expectations equilibrium (e.g., Evans and Honkapohja 2001; 2011). However, as I shall illustrate, in this case, for low values of π, such as $\pi = \frac{1}{2}$ as used in the main calibration, there does not exist any stationary REE; but adaptive learning can still lead to truthful reporting and a stationary price function.

2.4.3 Bellman equation

I assume that investors' expectations are observable to the manager.[17] Therefore, the manager observes the state variables θ_t, x_t and $\hat{\pi}_t$, and makes reporting decision y_t to maximize her expected utility,

$$\max_{y_t \in \{\theta_H, \theta_L\}} E_t\Big[\sum_{s=0}^{\infty} \beta^s \ln(p_{t+s}) | (\theta_t, x_{t-1}, \hat{\pi}_t)\Big] \tag{2.4.4}$$

The manager's optimization problem can be formulated recursively. Given initial state $(\theta_0, y_0, x_0, \hat{\pi}_0)$, the manager's value function for each period t is given by the following

[17] The assumption that some agents' beliefs form the basis of informationally-advantaged agents' decision is also seen in other economic settings. For example, Evans and Honkapohja (2003) study a central bank's policy based on agents' expectations.

Bellman equation,

$$v_t(y_{t-1}, \theta_t, x_{t-1}, \hat{\pi}_{t-1}) = \max_{y_t \in \{\theta_H, \theta_L\}} \left\{ \ln(p_t(y_t, \hat{\pi}_t)) + \beta E_t[v_{t+1}(y_t, \theta_{t+1}, x_t, \hat{\pi}_t)] \right\} \quad (2.4.5)$$

where $v_t(\cdot)$ denotes the value function in period t. The laws of motion are summarized as follows.

(i) The state variables in the initial period are given as $(y_0, \theta_0, x_0, \hat{\pi}_0)$;

(ii) Real earnings are governed by an exogenous stochastic process as described in Section 2.1;

(iii) Reported earnings are determined by the optimal decision rule $y_t = y_t(y_{t-1}, \theta_t, x_{t-1}, \hat{\pi}_{t-1})$;

(iv) Accruals are given by $a_t = y_t - \theta_t$;

(v) The law of motion for accumulated accruals x_t is $x_t = x_{t-1} + a_t$ and $x_t \in \{-A, 0, A\}$;

(vi) The law of motion for the probability assessment $\hat{\pi}_t$ is given by (2.4.1);

(vii) The price function $p(\cdot)$ is given by (2.4.2). The Bellman equation can be rewritten as 12 univariate equations.

There are four state variables, $y_{t-1}, \theta_t, x_{t-1}$, and $\hat{\pi}_{t-1}$. However, because the first three are discrete, the Bellman equation for each period can be written as 12 univariate functions of $\hat{pi}_{t-1}$, $v_{ijk}(\hat{\pi}_{t-1})$, where the subscripts i, j and k correspond to $y_{t-1} \in \{\theta_H, \theta_L\}$, $\theta_t \in \{\theta_H, \theta_L\}$, and $x_{t-1} \in \{-A, 0, A\}$, respectively, and $0 \leq \hat{\pi}_{t-1} \leq 1$. For example, $v_{t,HL+}$ is the value function when $y_{t-1} = \theta_H$, $\theta_t = \theta_L$ and $x_{t-1} = A$. Based on condition (2.4.5) and the laws of motion, it is straightforward to obtain an alternative representation of Bellman equation in terms of $v_{t,ijk}(\hat{\pi}_{t-1})$.

Proposition 2.3. For each period t, the Bellman equation can be represented by 12 univariate functions, $v_{t,ijk}(\hat{\pi}_{t-1})$, where $i, j \in \{H, L\}$, $k \in \{-, 0, +\}$, and $0 \leq \hat{\pi}_{t-1} \leq 1$. The 12 functions are defined by condition (2.9.8) in the appendix.

When real earnings are high (low) and accumulated accruals are negative (positive), the manager is constrained by the balance sheet, and the only permissible decision is to truthfully report earnings. For example, $v_{t,HH-}(\hat{\pi}_{t-1})$ is simply the utility from truthful reporting, i.e.,

$$v_{t,HH-}(\hat{\pi}_{t-1}) = \ln\Big(p\Big(\theta_H, (1-\frac{1}{t})\hat{\pi}_{t-1}+\frac{1}{t}\Big)\Big)+\beta\Big[\pi v_{t+1,HH-}\Big((1-\frac{1}{t})\hat{\pi}_{t-1}+\frac{1}{t}\Big)+(1-\pi)v_{t+1,HL-}\Big((1-\frac{1}{t})\hat{\pi}_{t-1}+\frac{1}{t}\Big)\Big] \tag{2.4.6}$$

For the other eight scenarios, the manager is not constrained by the balance sheet, and may truthfully report or manage earnings. For example, $v_{t,HH0}(\hat{\pi}_{t-1})$ is the higher of the utility derived from truthful reporting and the utility from earnings management, i.e.,

$$\begin{aligned} &v_{t,HH0}(\hat{\pi}_{t-1})\\ &= \max\Big\{\ln\Big(p\Big(\theta_H, (1-\frac{1}{t})\hat{\pi}_{t-1}+\frac{1}{t}\Big)\Big)+\beta\Big[\pi v_{t+1,HH0}\Big((1-\frac{1}{t})\hat{\pi}_{t-1}+\frac{1}{t}\Big)+(1-\pi)v_{t+1,HL0}\Big((1-\frac{1}{t})\hat{\pi}_{t-1}+\frac{1}{t}\Big)\Big],\\ &\ln\Big(p\Big(\theta_L, (1-\frac{1}{t})\hat{\pi}_{t-1}\Big)\Big)+\beta\Big[\pi v_{t+1,LH-}\Big((1-\frac{1}{t})\hat{\pi}_{t-1}\Big)+(1-\pi)v_{t+1,LL-}\Big((1-\frac{1}{t})\hat{\pi}_{t-1}\Big)\Big]\Big\} \end{aligned} \tag{2.4.7}$$

Based on the Contraction Mapping Theorem (Stokey et al. 1989, p. 50), I solve for the value functions $v_{t,ijk}(\cdot)$ and the associated decision rules $y_{t,ijk}(\cdot)$ as fix points of an iterative algorithm.[18]

2.4.4 Reporting decisions

For calibration purpose, I focus on the real earnings process with $\pi = 0.5$, i.e., a Bernoulli process. Alternative processes with $\pi > 0.5$ or $\pi < 0.5$ will be deferred to Section 2.6.5. Figures 2.1(a)-(d) report the decision rules $y_{t,ijk}(\hat{\pi}_{t-1})$, in an example period $t = 3$, for a baseline calibration of other exogenous parameters: $(\theta_H, \theta_L, \beta, \delta)$=(3, 1, 0.95, 0.06). I group decision rules by last period's reported earnings y_{t-1} and current period real earnings θ_t. The decision rules illustrate the conditions under which the manager chooses to truthfully report or manage earnings. As discussed above, four of the 12 optimal

[18]For brevity, I refer interested readers to the internet appendix for details of the algorithm.

decision rules are always to truthfully report (y_{HH-}, y_{HL+}, y_{LH-}, and y_{LL+}). However, for other scenarios, the manager follows a threshold rule conditional on $\hat{\pi}_{t-1}$, i.e., last period's investor belief of the earnings process. The value functions (not reported for brevity) correspond to the decision rules, and exhibit kinks where managers switch from truthful reporting to earnings management.

The threshold rules for misreporting warrant some discussion. When real earnings are high (Figures 2.1(a) and 2.1(c)), the manager tends to manage earnings if $\hat{\pi}_{t-1}$ is lower than a threshold level; and the threshold is lower (i.e., the manager is less likely to manage earnings) when the manager is more constrained by the balance sheet. For example, as Figure 2.1(a) shows, the threshold when accumulated accruals are zero (y_{HH0}) is lower than the threshold when accumulated accruals are positive (y_{HH+}), because lower accumulated accruals constrain downward earnings management more. When real earnings are low (Figures 2.1(b) and 2.1(d)), the manager tends to manage earnings when $\hat{\pi}_{t-1}$ is higher than a threshold level, and the threshold is higher (i.e., the manager is less likely to manage earnings) when the manager is more constrained by the balance sheet.

Intuitively, the threshold decision rules are determined by the tradeoff between the benefit and cost of earnings management. When real earnings are high, the benefit of misreporting (downward earnings management) is a build-up of earnings reserve for later periods, while the cost is the fact that investors may view deflated earnings as bad news that has implications for future periods. The cost is lower when investors' estimate of the "earnings persistence" ($\hat{\pi}_{t-1}$) is lower, and that is when the manager finds misreporting more appealing.[19] Analogously, when real earnings are low, the benefit of misreporting (upward earnings management) is to walk up investors' expectations of future earnings,

[19] To be precise, the argument of the value functions and decision rules is the probability assessment from last period, $\hat{\pi}_{t-1}$, instead of the probability assessment of this period, $\hat{\pi}_t$. However, the intuition remains unaffected.

while the cost is higher accumulated accruals that may constrain future misreporting. The benefit is higher when investors' estimate of the "persistence" ($\hat{\pi}_{t-1}$) is high, and that is when firm manager finds earnings inflation more rewarding.

As reported in Figures 2.1(e)-(f),[20] the thresholds decrease over time, and become 0 after about 20 periods. This confirms the convergence result as stated in Lemma 2.3.

2.5 Bayesian Learning with Regime-shifting Beliefs

2.5.1 Regime-shifting beliefs

Investors may perceive earnings as following certain patterns which in fact do not exist, and use the hypothetical patterns to predict future earnings. In this section, I examine the expectations scheme in which investors update their beliefs in a Bayesian fashion, but based on misspecified models.[21]

This perspective is also taken by a growing literature on under-parameterized forecasting models in various settings, in which some form of model misspecification is exogenously imposed on agents' beliefs (e.g., Barberis et al. 1998; Hong, Stein and Yu 2007; Branch and Evans 2010). While in these studies agents make decisions based on exogenously endowed streams of data, in my study agents (investors) form expectations on endogenous data (reported earnings). Thereby, this study is distinct from previous theories of misspecified learning by modeling the active decisions of the producer of the information based on which investors form beliefs. Simply put, what if the data source based on which one agent (say, agent A) forms beliefs is manipulated by another agent (say, agent B), and the manipulation is driven by the beliefs of agent A?

[20] Figures 2.1(e)-(f) report the variation in misreporting thresholds for the case of $\pi = 0.75$, because the case of $\pi = 0.5$ produces two figures that are identical.

[21] The term "misspecified" is used in a strong sense. In adaptive learning, investors necessarily have misspecified beliefs about the process of reported earnings in the process of learning, but this form of model misspecification is the norm of adaptive learning, and is not referred to as "incorrect" in this paper.

A theoretic critique to the plausibility of "perpetually misspecified beliefs" is, why cannot investors learn to detect and discard their specifications as data accrue? There are three conceivable responses to this critique. First, incorrect beliefs can be entrenched and sticky. Second, the horizon of investors' decision making may not be long enough to detect specification errors. Third, regime-shifting beliefs have garnered support from psychological and experimental studies. For example, the regime-shifting paradigm has been shown to be consistent with the conservatism and representativeness heuristics studied in decision making theories (e.g., Griffin and Tversky 1992), and experimental evidence on MBA students' earnings predicting behavior (e.g., Bloomfield and Hales 2002). From an empirical perspective, regime-shifting beliefs might be less biased than postulated in theories. At least in asset pricing, there is growing evidence of multiple regimes of returns of volatilities (e.g., Guidolin and Timmermann 2007).

To model regime-shifting beliefs, I follow Barberis et al. (1998) and assume that investors perceive earnings as governed by one of the two regimes: a mean-reverting regime (Regime 1) and a persisting regime (Regime 2). The two regimes differ by the transition probabilities between the two earnings levels, θ_H and θ_L. The mean-reverting regime is characterized by $\Pr(y_{t+1} = \theta_i | y_t = \theta_i) = 1 - \pi_H$ and $\Pr(y_{t+1} = \theta_j | y_t = \theta_i) = \pi_H$, where $i, j \in \{H, L\}$, $i \neq j$, and $\pi_H \geq 0.5$. In contrast, the persisting regime is characterized by $\Pr(y_{t+1} = \theta_i | y_t = \theta_i) = \pi_H$ and $\Pr(y_{t+1} = \theta_j | y_t = \theta_i) = 1 - \pi_H$. If $\pi_H = 0.5$, the two regimes are identical. Therefore, π_H captures the dissimilarity between the two regimes or "regime heterogeneity".

The regime in period t is denoted as $m_t \in \{1, 2\}$. Investors believe that the transition from one regime to the other is described by the following Markov process,

$$\Pr(m_{t+1}|m_t) = \begin{bmatrix} 1-\lambda_1 & \lambda_1 \\ \lambda_2 & 1-\lambda_2 \end{bmatrix}$$

Investors update their beliefs about the probability of the current regime in a Bayesian manner. Specifically, let $q_t \equiv \Pr(m_t = 1|(y_t, y_{t-1}, q_{t-1}))$,

$$q_t = \frac{[(1-\lambda_1)q_{t-1} + \lambda_2(1-q_{t-1})]\Pr(y_t|m_t = 1, y_{t-1})}{[(1-\lambda_1)q_{t-1} + \lambda_2(1-q_{t-1})]\Pr(y_t|m_t = 1, y_{t-1}) + [\lambda_1 q_{t-1} + (1-\lambda_2)(1-q_{t-1})]\Pr(y_t|m_t = 2, y_{t-1})} \tag{2.5.1}$$

The full-fledged expression of q_t is given by Expression 2 in by conditions (2.9.9) and (2.9.10). The price function is given by the following lemma.

Lemma 2.5. Given current period reported earnings y_t, and investors' probability assessment q_t of the current regime being mean-reverting, the stock price is

$$p(y_t, q_t) = \frac{\theta_H + \theta_L}{2\delta} + \frac{y_t - (\theta_H + \theta_L)/2}{1+\delta}\gamma_0'\Big[I - \frac{Q}{1+\delta}\Big]^{-1} Q(\gamma_1 + \gamma_2 q_t) \tag{2.5.2}$$

where γ_0, γ_1, γ_2, and Q are given in Lemma 2.6 presented in the appendix (Section 2.9.1).

2.5.2 Bellman equation

Given initial state $(\theta_0, y_0, x_0, q_0)$, the manager's value function is formulated recursively as the following Bellman equation,

$$v(y_{t-1}, \theta_t, x_{t-1}, q_{t-1}) = \max_{y_t \in \{\theta_H, \theta_L\}} \Big\{ \ln\Big(p(y_t, q_t)\Big) + \beta E_t[v(y_t, \theta_{t+1}, x_t, q_t)] \Big\} \tag{2.5.3}$$

(i) The state variables in the initial period are given as $(y_0, \theta_0, x_0, q_0)$;

(ii) Real earnings are governed by an exogenous stochastic process as described in Section 2.2.1;

(iii) Reported earnings are determined by the optimal decision rule $y_t = y(y_{t-1}, \theta_t, x_{t-1}, q_{t-1})$;

(iv) Accruals are given by $a_t = y_t - \theta_t$;

(v) The law of motion for accumulated accruals x_t is $x_t = x_{t-1} + a_t$ and $x_t \in \{-A, 0, A\}$;

(vi) The law of motion for the probability assessment q_t is $q_t = q^i(q_{t-1})$, $i = 1, 2$, where the expressions and conditions of $q^i(\cdot)$ are given by conditions (2.9.9) and (2.9.10);

(vii) The price function $p(\cdot)$ is given by condition (2.5.2).

Similar to the case of adaptive learning, it is easy to rewrite the Bellman equation in terms of $v_{ijk}(q_{t-1})$.

Proposition 2.4. The Bellman equation can be represented by 12 univariate functions, $v_{ijk}(q_{t-1})$, where $i, j \in \{H, L\}$, $k \in \{-, 0, +\}$, and $0 \leq q_{t-1} \leq 1$. The 12 equations are defined by condition (2.9.15) (Expression 3) in the appendix.

2.5.3 Reporting decisions

The dynamic programming problem can be solved by value function iteration. Unlike adaptive learning under which value functions and decision rules vary from period to period due to the time-indexed investor beliefs, under regime-shifting the value functions and decision rules remain the same through time.

Figure 2.2 reports the decision rules for a baseline calibration: (θ_H, θ_L, β, δ, π_H, λ_1, λ_2)=(3, 1, 0.95, 0.06, 0.75, 0.1, 0.3). I calibrate λ_1 and λ_2 to small values (0.1 and 0.3), because it is usually assumed that regime shifts are rare (e.g., Barberis et al. 1998). Decision rules are characterized by threshold rules based on q_{t-1}, i.e., last period's probability assessment of the regime being mean-reverting. When real earnings are high (Figures 2.2(a) and 2.2(c)), the manager tends to manage earnings if q_{t-1} is higher than a threshold level; the threshold is higher when the accumulated accruals approach the limit. When real earnings are low (Figures 2.2(b) and 2.2(d)), the manager tends to manage earnings when q_{t-1} is lower than a threshold level, and the threshold is lower when the constraint on accumulated accruals is more binding.

Intuitively, the manager weighs the capital market benefit (cost) of earnings management against the cost (benefit) associated with depleting (increasing) earnings reserve that affects her ability in the future to manipulate earnings. The capital market consequence of earnings management is now captured by q_{t-1}, which indicates the likelihood of the earnings process being mean-reverting as perceived by the capital market.

2.6 The Dynamics of Earnings and Asset Prices

Unlike static or two-period models of earnings management, this model offers empirical predictions on the dynamics of earnings and stock prices. Based on manager's optimal reporting strategies and investor beliefs as characterized above, I simulate histories of earnings and stock price for each of the three expectations scheme. I then study the time-series properties of earnings and prices, the steady-state distribution of earnings and accruals, and accounting-based return regularities.

2.6.1 Earnings and prices

Based on the decision rules, I can simulate the history of financial reporting and stock prices. Because rational expectations reflect a "result" of learning, the stationary REE is not directly comparable to the other two forms of expectations schemes.[22] Thereby, I focus on the earnings management and asset price dynamics under adaptive learning and regime-shifting beliefs.

[22]Under a truthful reporting rational expectations equilibrium, real earnings and reported earnings processes coincide. It could be still of interest to study the dynamics under a stationary REE with earnings management.

Adaptive learning

In order to simulate the model, I need to initialize the state variables in the first period, $(y_0, \theta_0, x_0, \hat{\pi}_0)$. Without loss of generality, suppose that in period 0, the manager truthfully reports earnings ($y_0 = \theta_H$), there are no accumulated accruals ($x_0 = 0$), and investors' prior is $\hat{\pi}_0 = 0.5$. Real earnings are generated by a Bernoulli process with probability 0.5.

Based on the initial state, history of real earnings, laws of motion for state variables, and optimal decision rules, I can simulate reported earnings as $y_t = y(y_{t-1}, \theta_t, x_{t-1}, \hat{\pi}_{t-1})$, the accruals component of earnings as $a_t = y_t - \theta_t$, accumulated accruals as $x_t = x_{t-1} + a_t$, and investor belief $\hat{\pi}_t$, stock price $p(y_t, \hat{\pi}_t)$, for $t = 1, 2, ...$ For comparison purpose, I also simulate hypothetical investor beliefs and hypothetical prices based on real earnings (as if there were no earnings management and real earnings were observed by investors),

$$p_t^{\text{real}} = p(\theta_t, \hat{\pi}_t^{\text{real}},) \tag{2.6.1}$$

where the price function is given by condition (2.4.2), and $\hat{\pi}_t^{\text{real}}$ is determined by analogs of (2.4.1) with y_t replaced by θ_t.

Figure 2.3 reports the simulation results. As Figure 2.3(a) shows, earnings management exists in early periods, but disappears after 10 periods as investors' perceived law of motion converges to the actual law of motion. Figure 2.3(c) illustrates the evolution of investor beliefs over time. Apparently, investors' beliefs gradually converge to the actual law of motion for reported earnings, even though the latter is also a result of investor expectations. The history of stock price mirrors that of investor beliefs. As Figure 2.3(e) shows, as investor beliefs converge to actual law of motion, price becomes less volatile and reflects only earnings shocks. As a result, in the long run, the state vector s_t can only take two values (θ_H, A) and (θ_L, A), with equal probabilities, as will be illustrated in Figure 2.5(a).

Regime-shifting beliefs

Under regime-shifting beliefs, earnings management is not disciplined away by investor learning even in the long run. Figure 2.3(b) reports the histories of reported earnings and real earnings. There is considerable income smoothing, as indicated by more and longer sequences of consecutive same-level reported earnings than real earnings. Figures 2.3(d) and 2.3(f) report investor beliefs and stock prices. Because of income smoothing, investor beliefs and price movements exhibit less frequent oscillations relative to the hypothetical price without earnings management. The probability distribution of the state vector s_t is illustrated in Figure 2.5(b). When real earnings are high, it is more likely to have zero accumulated accruals (with probability 0.235) than negative or positive accumulated accruals (with probabilities 0.120 and 0.145, respectively). In contrast, when real earnings are low, it is more likely to have positive accumulated accruals (with probability 0.235) than negative or zero accumulated accruals (with probabilities 0.145 and 0.120, respectively).

I then study how earnings management activities vary with investor beliefs of the regime heterogeneity, as measured by π_H.[23] Figure 2.4(a) reports the comparative statics of earnings management with respect to investor sentiment. Earnings management, as measured by the fractions of periods with upward or downward earnings management ("up" and "down"), exhibits an inverted-U-shaped relationship with π_H for $\pi_H \in [0.5, 1]$. In other words, earnings management is most aggressive when the two regimes perceived by investors are moderately different. To understand this result, note that manager has less incentives for misreporting when $\pi_H \to 0.5$ and when $\pi_H \to 1$. When $\pi_H \to 0.5$, the

[23] For comparative statics, I simulate a relatively long history for each parameter value. The large number of periods is chosen to reduce estimation error rather than for approximation of reality. In fact, the number of periods does not matter for the dynamics of earnings and prices. Unreported results show that simulation of 50 periods yields very similar but coarser patterns.

two regimes perceived by investors become Bernoulli processes, and the level of current earnings has no implications for future earnings. Therefore, the manager has no incentive to manage earnings. When $\pi_H \to 1$, investors perceive the earnings process to be either highly mean-reverting or highly persisting, and investors' probability assessment of the earnings regime being mean-reverting is $q_t \to 0$ if $y_t = y_{t-1}$, and $q_t \to 1$ if $y_t \neq y_{t-1}$. In this case, investors' probability assessment is highly volatile, swinging between 0 and 1, and the benefit of earnings management is short-lived and offset by immediate adverse consequences.[24] In comparison, intermediate values of π_H provide greater incentives to manage earnings.

I also study the role of regime heterogeneity on value relevance, mean return, and Sharpe ratio. Value relevance is calculated as the coefficient on y_t from the following firm-specific regression,

$$p_t = \beta_0 + \beta_1 y_t + \epsilon_t \tag{2.6.2}$$

Mean return is the time-series average of returns, and Sharpe ratio is mean return divided by the standard deviation of returns. The three metrics are also calculated based on real earnings and hypothetical prices. For example, the value relevance of real earnings in the hypothetical case is the coefficient on θ_t from the following regression,

$$p_t^{\text{real}} = \beta_0 + \beta_1 \theta_t + \epsilon_t \tag{2.6.3}$$

Figures 2.4(b)-(d) report the results for the three metrics, respectively. Value relevance exhibits an asymmetric V-shaped relationship with π_H, while mean return and Sharpe ratio increase with π_H. These patterns also exist in the case without earnings management.

[24] For example, if last period's reported earnings are high and this period's real earnings are low, managing earnings upward makes investors believe that earnings are persistent, but the benefit is short-lived because investor's belief is reversed once next period's real earnings are low, due to the balance sheet constraint.

However, in the presence of managerial discretion, value relevance is higher, mean return is lower, compared with the case without earnings management. The case for Sharpe ratio is more interesting: Managerial opportunism increases Sharpe ratio when investor misbelief is less severe, but decreases it when investors believe in strong regime heterogeneity.

2.6.2 Accounting-based return regularities

Predicting returns

To understand how stock returns are determined by earnings reports and investor expectations, I derive expressions of price changes. Under rational expectations, based on Lemma 2.1, it is easy to show that the change in price from t to $t+1$ equals

$$p(y_{t+1}) - p(y_t) = \frac{1+\delta}{2(1-\pi)+\delta}(y_{t+1} - y_t) \tag{2.6.4}$$

where $y_t \in \{\theta_H, \theta_L\}$. Therefore, stock returns only reflect the news in earnings reports, which are discretionarily determined by the manager.

Under adaptive learning and regime-shifting beliefs, price changes reflect changes in earnings reports as well as changes in investor beliefs.[25] Under adaptive learning, based on Lemma 2.2, the price change equals

$$p(y_{t+1}, \hat{\pi}_{t+1}) - p(y_t, \hat{\pi}_t) = \frac{1+\delta}{\delta}\left[\frac{(1-\hat{\pi}_{t+1})(\theta_H+\theta_L)+\delta y_{t+1}}{2(1-\hat{\pi}_{t+1})+\delta} - \frac{(1-\hat{\pi}_t)(\theta_H+\theta_L)+\delta y_t}{2(1-\hat{\pi}_t)+\delta}\right] \tag{2.6.5}$$

where $\hat{\pi}_t$ is the investor belief in period t. As investors have more earnings reports, $\lim_{t\to\infty}\hat{\pi}_t \to \pi$, and their perceived law of motion for reported earnings approaches the

[25] A key difference from price changes in models with exogenous dividends and investor learning, such as Lewellen and Shanken (2002), is that earnings news are jointly determined with price.

actual law of motion, and the price change will converge to the case of stationary REE,

$$\lim_{t\to\infty} [p(y_{t+1}, \hat{\pi}_{t+1}) - p(y_t, \hat{\pi}_t)] = \frac{1+\delta}{2(1-\pi)+\delta}(y_{t+1} - y_t) \tag{2.6.6}$$

Under regime-shifting beliefs, based on Lemma 2.5, the price change equals

$$p(y_{t+1}, q_{t+1}) - p(y_t, q_t) = \frac{1}{1+\delta}\gamma_0'\Big[I - \frac{Q}{1+\delta}\Big]^{-1} Q\Big[\gamma_1(y_{t+1}-y_t) + \gamma_2\big[q_{t+1}y_{t+1} - q_t y_t - \frac{\theta_H+\theta_L}{2}(q_{t+1}-q_t)\big]\Big] \tag{2.6.7}$$

where γ_0, γ_1, γ_2, and Q are given in Lemma 2.6. As q_t never converges to a constant, the impact of investor beliefs on stock returns will persist in the long run.

Underreaction, overreaction, and the accruals anomaly

The dynamic setting can be used to analyze the interaction between earnings/accruals and stock returns when earnings are opportunistically reported. In particular, I am interested in earnings- and accruals-based return regularities, such as short-term underreaction and long-term overreaction to earnings, and the accruals anomaly. "One period" in the model can be interpreted as a year, and reported earnings as the annual earnings reported on 10-K filings.[26]

It is widely documented that stock prices exhibit delayed adjustment to earnings news in the short-term (e.g., Bernard and Thomas 1989), but tend to "overreact" to extreme news (e.g., DeBondt and Thaler 1985, 1987; Zarowin 1989). Prior literature has proposed a plethora of explanations for underreaction and overreaction (e.g., Barberis et al. 1998; Hong and Stein 1999). In particular, Barberis et al. (1998) show that for a wide range of parameter values, regime-shifting beliefs can generate both underreaction and overreaction.

[26]The proper interpretation of the model horizon also depends on the choice of investor discount rates. As there are many simplifying assumptions that make model output not comparable with real world data, I am mainly interested in qualitative patterns rather than actual magnitudes of returns.

Another important accounting-based anomaly is that the accruals component of earnings negatively predicts future returns (Sloan 1996). Researchers have proposed various explanations for the accruals anomaly, including investor fixation on accruals (e.g., Sloan 1996), earnings persistence (e.g., Richardson et al. 2005), and investment (e.g., Wu et al. 2010). By introducing earnings shifting and accruals to an intertemporal model of investor learning, my setting may potentially provide new insights on factors underlying the return predictability based on accruals.[27]

To study underreaction and overreaction to earnings, I simulate 1000 independent histories (which represent idiosyncratic firms) for 50 periods, and form portfolios based on the previous n ($n = 1, ..., 5$) periods of earnings. For each period with sufficient data, I form a zero-investment portfolio by buying all firms with positive earnings in each of the previous n periods, and shorting all firms with negative earnings in each of the n periods. I use the equal-weighted return as portfolio return, and report the average return to the portfolios formed on all n periods. If both short-term underreaction and long-term overreaction exist, I should observe a downward sloping hedge return as n increases, with positive return for small n (e.g., $n = 1$) and negative return for large n ($n = 5$).

To study the accruals anomaly, I simulate 1000 idiosyncratic firms for 50 periods, and form portfolios based on the level of accruals, $a_t = \{-A, 0, A\}$. Specifically, for each period, I form a zero-investment portfolio by buying all firms with negative accruals (i.e., $a_t = -A$) and shorting all firms with positive accruals (i.e., $a_t = A$). Thereby, I take long position in firms with deflated earnings and short position in firms with inflated earnings. I use the equal-weighted returns as portfolio return, and report the average

[27] In this model, accruals are not public information. Extreme as this assumption appears, it is not entirely an imagined scenario. In the real world, even if investors have access to accruals information, there is a large body of evidence that investors fixate on earnings and pay insufficient attention to accruals. As long as the accruals anomaly is driven and perpetuated by this group of investors, the insights from our model are relevant for studying the accruals anomaly. When accruals are not observable, any "anomalous" returns associated with accruals would not violate semi-strong form market efficiency; nonetheless, I continue to use the terms "anomalous" and "anomaly" for expositional convenience.

return to all portfolios for several periods subsequent to portfolio formation. The existence of accruals anomaly would be indicated by a positive return to the hedge portfolio over some forecasting horizon.

Adaptive learning

Under adaptive learning, investors' perceived law of motion is adjusted each period until it asymptotically converges to the actual law of motion. After a large number of periods, adaptive learning approaches a stationary equilibrium characterized by truthful reporting and positive accumulated accruals. Therefore, I focus on the short-run learning process during which there is still earnings management. Specifically, I calculate accounting-based anomalies based on data from the first 50 periods.

There is no underreaction to earnings news, but there is substantial overreaction, as illustrated in Figure 2.5(c). This is because recursive learning leads investors to put more weight on most recent earnings news: Recent earnings news not only forms the basis of prediction, but also influences investor beliefs. This is in line with the finding of Lewellen and Shanken (2002) that estimation errors tend to cause mean reversion in stock prices.

There is also significant return predictability based on accruals. Firms that have just experienced negative accruals outperform firms that have just experienced positive accruals, generating an accruals anomaly in the magnitude of 4%-5% in the first two periods subsequent to portfolio formation. The hedge return dissipates to around 1% after three periods, and then becomes insignificantly negative.

Regime-shifting beliefs

Under regime-shifting beliefs, there exist both underreaction and overreaction to earnings news. The last column of Table 2.1 and Figure 2.5(d) report the returns to the hedge portfolio: The average return following one period of positive earnings is higher than the average return following one period of negative earnings, with a difference of 0.869%, indicating underreaction to earnings. When the sequence of consecutive positive/negative earnings based on which portfolios are formed gets longer (when n is larger), the hedge return monotonically decreases, and becomes negative when $n \geq 4$. The average return following five periods of positive earnings is 0.351% lower than the average return following five periods of negative earnings, indicating substantial overreaction.

Accruals anomaly is also evident under regime-shifting beliefs. The last three columns of Table 2.2 and Figure 2.5(f) report the returns to the hedge portfolio and its two components: firms with negative accruals and firms with positive accruals. Negative-accruals firms underperform positive-accruals firms in the first period subsequent to portfolio formation. This is because the accruals anomaly is subsumed by the delayed reaction to inflated earnings, as documented in the previous section (the magnitude of return differential, 0.568%, is smaller to the hedge return to underreaction strategy when $n = 1$, 0.864). But the comparison is soon flipped as I hold the portfolio into future periods. For the second and third periods after portfolio formation, negative-accruals firms outperform positive-accruals firms by 0.353% and 0.457%, which corresponds to the widely documented accruals anomaly. After three periods, the hedge return becomes insignificant.

I then study how regime heterogeneity influences the anomalies. I find that greater regime heterogeneity is associated with more severe investor misreactions. As reported in Figure 2.4(e), the plot of hedge returns becomes steeper when π_H is increased to 0.95, and it becomes virtually flat when π_H is reduced to 0.60. I also find that the trading strategy that exploits the accruals anomaly is most profitable for highly dissimilar regimes ($\pi_H = 0.95$), with a second-period hedge return of 3.568%. In contrast, when the two regimes are similar to each other and to the underlying earnings process ($\pi_H = 0.60$ is close to 0.50), the hedge returns for the second and third periods are merely 0.119% and 0.093%, respectively. Figure 2.4(f) provides a graphical representation of the comparisons of hedge returns for different values of regime heterogeneity.

This finding sheds light on the role of investor beliefs in explaining the accruals anomaly. When investors believe in switching regimes that deviate substantially from the underlying earnings process, their expectations and stock prices are prone to manipulation. As their misperceptions are eventually corrected or reversed, sizable anomalous returns can be generated. The return predictability based on accruals is just a ramification of this process.

2.6.3 Descriptive validity of expectations schemes

A plethora of rational and behavioral theories have been proposed to explain return regularities. Relying on models with exogenous earnings/dividend processes, researchers have also compared the performance of various theories of investor beliefs in fitting the real world data. For example, Brav and Heaton (2002) show that the predictions of a theory built on investor irrationality and a theory built on rational structural uncertainty are actually hard to distinguish. This statement is also a good summary of the current consensus in this literature.

A pitfall of this literature is that it ignores a simultaneity problem: In reality, earnings or dividend processes reflect managerial decisions which are jointly determined with investor expectations and asset prices. Therefore, the findings based on models with exogenous earnings might be limited. In contrast, this paper incorporates managerial discretion, and by doing so, provides a richer setting for testing various hypotheses on investor beliefs. In addition, by explicitly modeling accounting accruals, this model can be used to compare how well expectations schemes explain accruals-related phenomenon. I find that regime-shifting beliefs generate various return regularities that correspond to those documented in empirical studies, while adaptive learning generates larger amount of accruals anomaly.

2.6.4 The impact of managerial discretion

With a model of "self-referential" nature, managerial opportunism is jointly determined with investor expectations. It is intriguing to investigate whether managerial discretion (or earnings management) amplifies or mitigates stock market misreactions to earnings news. To obtain benchmark anomalous returns in a world devoid of managerial discretion, I sort hypothetical stock returns (as if investors were able to observe the real earnings) on real, unmanaged earnings.[28]

As reported in Table 2.2, the hedge returns based on unmanaged earnings are larger in magnitude than the case with earnings management. The hedge returns decrease from 1.038% for $n = 1$ to -0.387% for $n = 5$. In other words, in the presence of managerial discretion, a trading strategy based on short-term underreaction and long-term overreaction is more profitable both in the short-term and in the long-term. One possible explanation is that discretionary earnings smoothing dampens the oscillation of investor beliefs between the two regimes, and thereby reduces the magnitudes of misreactions to earnings, because each regime shift represents a (possibly excessive) corrective adjustment of beliefs.

2.6.5 Underlying earnings processes

In the rational expectations literature, researchers following Muth (1961) have examined the role of the stochastic process in expectations formation. In this section, I study the impact of underlying earnings process on investor beliefs and financial reporting decisions.

Under rational expectations, the underlying earnings process determines which sta-

[28]The results on unmanaged earnings are consistent with the main findings of Barberis et al. (1998), although the magnitudes are different due to different pricing functions.

tionary REE holds; under the other two schemes, it may play a more intricate role. I illustrate this issue by revisiting the scheme of regime-shifting beliefs. In the main calibration, I have focused on the case in which real earnings are generated by a Bernoulli process with equal probabilities ($\pi = 0.50$). In this section, I study two alternative processes for real earnings: a mean-reverting process ($\pi = 0.25$) and a persisting process ($\pi = 0.75$)[29]. I examine whether the choice of real earnings process changes manager's reporting decisions and regularities in asset prices.

Adopting a different real earnings process may affect model outcomes in two ways. First, it changes the functional form of value functions and decision rules, which are functions of π (see, for example, Proposition 2.1). Second, the simulation paths are different due to different processes of real earnings, which are input of the decision rules and laws of motion for reported earnings and accruals. It does not, however, affect how investors update their beliefs, and thereby the price function stays the same as before.

Untabulated results show that the decision rules for mean-reverting and persisting real earnings are qualitatively the same with the case of Bernoulli real earnings, except that the threshold levels are different. I also conduct similar analysis on the dynamics of earnings and prices, and find that the impact of regime heterogeneity on earnings management and asset prices does not change qualitatively with the choice of real earnings process.

Accounting-based anomalies exist under both alternative processes, but the role of managerial discretion varies with real earnings process. Specifically, when real earnings are mean-reverting, stock returns underreact less and overreact less to managed earnings than to unmanaged earnings, same as the case of Bernoulli real earnings. However, when

[29] It is also possible to study an alternative setup where *real* earnings are generated by a regime-shifting process, which would provide more underpinning for investors' regime-shifting beliefs. However, adopting regime-shifting process for real earnings would introduce another state variable (the current regime that generates real earnings) without gaining substantial incremental insights. A regime-shifting process is essentially a Markov-modulated combination of two stochastic processes. Since neither mean-reverting real earnings nor persisting real earnings qualitatively change the misreporting decision rules, a combination of the two processes is also unlikely to change the decision rules.

real earnings are persisting, stock returns underreact more and overreact less to managed earnings than to unmanaged earnings.

2.7 Extensions

This paper provides a "minimalist" model of opportunistic financial reporting in the presence of investor learning. In this model, both real and reported earnings are binary, and investor beliefs are simply captured by two-state Markov processes. In this section, I discuss several directions to generalize or extend the current model.

(i) *Continuous earnings.* In the main model, earnings are discrete and only take two values. This setup facilitates an intuitive representation of the optimization problem while maintaining the richness of testable predictions. However, it would give rise to richer predictions if earnings can take continuous values. In Section 2.9.2, I provide an outline for formulating the manager's optimization problem when earnings can be reported in continuous values.

(ii) *Constraint on accumulated accruals.* Without loss of generality, I have assumed a tight constraint on income shifting: $-A \leq x_t \leq A$. In other words, the manager cannot inflate or deflate earnings for more than two consecutive periods. In reality, the manager may have more leeway in earnings management. The model can be easily modified to incorporate a more lenient constraint on accumulated accruals, such as $-2A \leq x_t \leq 2A$. Intuitively, this will lengthen the cycles of earnings manipulation/reversal, and the cycles of investor beliefs in the adaptive learning and regime-shifting cases.

(iii) *Heterogeneous beliefs.* In the current model, the capital market is represented by investors with homogenous beliefs (or equivalently a representative investor). This assumption is in line with Muth (1961) who observes that under rational expectations, the average of subjective beliefs of agents equals the actual model, while acknowledging that "there are considerable cross-sectional differences of opinion" in real-world expectations data (Muth 1961, p. 316). Therefore, it would be of interest to study heterogeneous expectations among investors. While I leave this extension for future research, it is worth noting that technically, a rational expectations equilibrium with belief heterogeneity can be defined in a similar way to dynamic stochastic general equilibrium models (e.g., Huggett 1993; or see Heathcote, Storesletten, and Violante 2009 for a review).

(iv) *Alternative parameterizations.* This setting can be applied to study the role of various innate factors in explaining the cross-sectional variations in earnings management and the resulting asset price dynamics. In alternative calibrations, I change the values of investor discount rate δ, manager time preference β, and earnings risk $\sigma \equiv \frac{2(\theta_H - \theta_L)}{\theta_H + \theta_L}$ (the standard deviation of real earnings scaled by the mean of real earnings). I also replace the logarithm utility function with a CRRA utility function of the following form,

$$u(c) = \frac{c^{1-\varphi}}{1-\varphi} \tag{2.7.1}$$

to study the impact of varying levels of managerial risk aversion (φ).

Results based on different parameterizations of the CRRA function indicate that the manager tends to manage earnings more often when investors have very long horizon (i.e., δ close to 0), and when earnings risk is high. However, earnings management does not vary with manager's time preference and risk aversion in an explicable way. These findings highlight the importance of capital market factors in explaining managerial opportunism.

I do not separately study the optimization problem of a finitely-lived manager because the manager's horizon in financial reporting has been effectively limited by the constraint on accumulated accruals. In addition, the comparative statics on β also confirms the intuition that adding another degree of freedom in managerial horizon does not generate additional variations.

2.8 Concluding Remarks

This paper proposes a dynamic model of earnings management and investor expectations. I solve for the manager's optimization problem, and study how managerial opportunism and investor beliefs are jointly determined. Under rational expectations, whether earnings management or truthful reporting constitutes an equilibrium depends on the properties of the underlying real earnings process. When investors do not have full knowledge and rational expectations, the manager's misreporting decision follows threshold rules conditioned on investors' beliefs, which could be investors' estimate of the earnings continuation rate (under adaptive learning) or their probability assessment of the current regime (under regime-shifting beliefs). Under adaptive learning, although earnings management exists in early periods when there is still substantial discrepancy between investors' perceived earnings process and the actual earnings process, over time it is eliminated through learning. Under Bayesian learning with regime-shifting beliefs, the perceived and actual earnings processes never converge to one another, and earnings management persists over time.

This paper sheds light on the dynamics of earnings management and asset prices in the presence of certain investor learning schemes. It provides many testable predictions on earnings management, time-series properties of earnings and prices, and accounting-based return regularities.[30] However, the emphasis of this paper is not to conduct a "horse

[30]Testing these predictions calls for firm specific measures of investor beliefs, which are absent in existing empirical studies. For example, to test predictions under the regime-shifting scheme, I will need cross-sectional measures of the regime-heterogeneity as perceived by investors. However, existing measures are mostly economy-wide measures of investor sentiment (e.g., the Composite Sentiment Index used in Baker and Wurgler (2006); the Michigan Consumer Confidence Index used in Bergman and Roychowdhury (2008)).

race" that compares the performance of each expectations scheme in explaining real world data. Instead, I focus on the implications of deviations from a full knowledge/rational expectations benchmark.

By examining the joint determination of opportunistic financial reporting decisions and investor expectations in a stochastic setting, this paper responds to Demski's (2004) call for research on accruals accounting from an "endogenous expectations" perspective.[31] In addition, a technical contribution of this paper is that it introduces recursive methods to the study of earnings management.[32] There are many conceivable extensions which I leave for future research. For example, in this paper I focus on the valuation role of accounting information, and there is no productive actions by the manager. Future research could examine how managers use real decisions in conjunction with reporting decisions to manage earnings under various investor expectations schemes.

[31] "This reliance on largely exogenous expectation structures, I think, needlessly limits the depth and boundaries of our teaching and research." (Demski 2004, p. 519)

[32] For standard references on recursive macroeconomic theories, see Stokey et al. (1989) and Ljungqvist and Sargent (2004). Also drawing on recursive macroeconomic theories, Kanodia (1980) studies a dynamic general equilibrium model of imperfectly informed capital market agents and corporate investment decisions. However, he focuses on the role of information structure on equilibrium outcome, instead of modeling earnings management by real or reporting decisions.

2.9 Appendix

2.9.1 Lengthy expressions and proofs

Proof of Lemma 2.1. Investor beliefs are given by

$$\Pr(y_{t+1} = \theta_j | y_t = \theta_j) = \frac{\sum_{k\in\{H,L\}} \Pr(\theta_t = \theta_k | y_t = \theta_j) M(k)}{\sum_{k\in\{H,L\}} \Pr(\theta_t = \theta_k | y_t = \theta_j) M(k) + \sum_{k\in\{H,L\}} \Pr(\theta_t = \theta_k | y_t = \theta_j) N(k)} \tag{2.9.1}$$

where

$$M(k) = \sum_{x_{t-1}\in X} \lambda(\theta_k, x_{t-1})[\pi \mathbf{1}_{y(\theta_k, x_{t-1}+\theta_j-\theta_k)=\theta_j} + (1-\pi)\mathbf{1}_{y(\theta_i, x_{t-1}+\theta_j-\theta_k)=\theta_j}] \tag{2.9.2}$$

$$N(k) = \sum_{x_{t-1}\in X} \lambda(\theta_k, x_{t-1})[\pi \mathbf{1}_{y(\theta_k, x_{t-1}+\theta_j-\theta_k)=\theta_i} + (1-\pi)\mathbf{1}_{y(\theta_i, x_{t-1}+\theta_j-\theta_k)=\theta_i}] \tag{2.9.3}$$

$$\Pr(\theta_t = \theta_k | y_t = \theta_j) = \frac{\sum_{x_{t-1}\in X} \lambda(\theta_k, x_{t-1}) \cdot \mathbf{1}_{y(\theta_k, x_{t-1})=\theta_j}}{\sum_{x_{t-1}\in X} \lambda(\theta_k, x_{t-1}) \cdot \mathbf{1}_{y(\theta_k, x_{t-1})=\theta_j} + \sum_{x_{t-1}\in X} \lambda(\theta_k, x_{t-1}) \cdot \mathbf{1}_{y(\theta_k, x_{t-1})=\theta_i}} \tag{2.9.4}$$

and $\Pr(y_{t+1} = \theta_i | y_t = \theta_j) = 1 - \Pr(y_{t+1} = \theta_j | y_t = \theta_j)$, where $\mathbf{1}_{y(\theta_j, x_{t-1})=\theta_j}$ is an indicator variable that takes the value 1 if $y(\theta_j, x_{t-1}) = \theta_j$, for $i, j \in \{H, L\}$ and $i \neq j$.

For any reporting strategy y of stationary REE, it is easy to show that investor belief is given by $\Pr(y_{t+1} = \theta_j | y_t = \theta_j) = \pi$, and $\Pr(y_{t+1} = \theta_i | y_t = \theta_j) = 1 - \pi$.

Therefore, the price function is

$$p(y_t) = \sum_{s=1}^{\infty} \left(\frac{1}{1+\delta}\right)^s E_t[y_{t+s}|y_t] = \left[\left(I - \frac{1}{1+\delta}\Pi\right)^{-1} \bar{\theta}\right]_j \tag{2.9.5}$$

where $\Pi = [\pi, 1-\pi; 1-\pi, \pi]$, $\bar{\theta} = [\theta_H, \theta_L]'$, I is an 2×2 identity matrix, and $j = 1$ if $y_t = \theta_H$, and $j = 2$ if $y_t = \theta_L$. Note that $\frac{1}{1+\delta} < 1$ ensures the convergence of $I + \frac{1}{1+\delta}\Pi + \frac{1}{(1+\delta)^2}\Pi^2 + ... = (I - \frac{1}{1+\delta}\Pi)^{-1}$.

Proof of Proposition 2.2. I focus on pure strategy stationary rational expectations equilibrium, and use a "guess and verify" approach to solve the model. It is easy to list all 16 possible candidates of decision rules ($16 = 2^4$, because the manager has flexibility of earnings management only in four out of six states: $\{y_{L-}, y_{H0}, y_{L0}, y_{H+}\} \in \Theta^4$). To illustrate how to verify whether some candidate equilibrium is indeed a valid equilibrium, consider the example of $[y_{H-}, y_{L-}, y_{H0}, y_{L0}, y_{H+}, y_{L+}] = [\theta_H, \theta_H, \theta_H, \theta_H, \theta_H, \theta_L]$.

(i) (Guess of reporting strategy.) Suppose that $[y_{H-}, y_{L-}, y_{H0}, y_{L0}, y_{H+}, y_{L+}] = [\theta_H, \theta_H, \theta_H, \theta_H, \theta_H, \theta_L]$ constitutes a stationary REE;

(ii) (Stationary distribution of state variables.) Under this reporting strategy, the stationarity condition requires that $\lambda'_{t+1} = \lambda'_t P$ while stationarity condition requires that $\lambda'_{t+1} = \lambda'_t$. Therefore,

$$[\lambda_{H+}\ \lambda_{L-}\ \lambda_{H0}\ \lambda_{L0}\ \lambda_{H+}\ \lambda_{L+}] = [\lambda_{H+}\ \lambda_{L-}\ \lambda_{H0}\ \lambda_{L0}\ \lambda_{H+}\ \lambda_{L+}] \begin{bmatrix} \pi & 0 & 0 & 0 & 0 & 0 \\ 1-\pi & 0 & 0 & 0 & 0 & 0 \\ 0 & 1-\pi & \pi & 0 & 0 & 0 \\ 0 & \pi & 1-\pi & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-\pi & \pi & 1-\pi \\ 0 & 0 & 0 & \pi & 1-\pi & \pi \end{bmatrix} \quad (2.9.6)$$

which together with $\lambda \cdot \mathbf{1} = 1$ yields

$$\lambda' = [\lambda_{H+}\ \ \lambda_{L-}\ \ \lambda_{H0}\ \ \lambda_{L0}\ \ \lambda_{H+}\ \ \lambda_{L+}] = [0\ \ 0\ \ 0\ \ 0\ \ \frac{1}{2}\ \ \frac{1}{2}] \quad (2.9.7)$$

(iii) (Incentive compatibility conditions.) The last step is to find conditions for model parameters that make this reporting strategy an incentive-compatible one. Specifically, I am interested in pinning down the range of π such that the equilibrium obtains. It is easy to show that no value of $\pi \in [0, 1]$ can support $y = (\theta_H, \theta_H, \theta_H, \theta_H, \theta_H, \theta_L)$ as an optimal strategy for the manager. Therefore, the conjectured strategy is ruled out as an equilibrium. The other 15 candidate strategies can be examined in a similar way.

Proof of Lemma 2.2. The proof is similar to that of Lemma 2.1, where Π is replaced by $\hat{\Pi} = [\hat{\pi}_t, 1-\hat{\pi}_t; 1-\hat{\pi}_t, \hat{\pi}_t]$ to account for the time-varying nature of investor beliefs.

Expression 1 (Proposition 2.3). The Bellman equations in Proposition 2.3 are given by 12 univariate functions of $\hat{\pi}_{t-1}$ where $0 \le \hat{\pi}_{t-1} \le 1$, $v_{ijk,t}(\hat{\pi}_{t-1})$, where $i, j \in \{H, L\}$, $k \in \{-, 0, +\}$, and $t = 1, 2, \ldots$.

$$
\begin{aligned}
v_{t,HH-}(\hat{\pi}_{t-1}) &= \ln\Big(p\Big(\theta_H, (1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big)\Big) + \beta\Big[\pi v_{t+1,HH-}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big) + (1-\pi)v_{t+1,HL-}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big)\Big] \\
v_{t,HH0}(\hat{\pi}_{t-1}) &= \max\Big\{ \ln\Big(p\Big(\theta_H, (1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big)\Big) + \beta\Big[\pi v_{t+1,HH0}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big) + (1-\pi)v_{t+1,HL0}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big)\Big], \\
&\qquad \ln\Big(p\Big(\theta_L, (1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big)\Big) + \beta\Big[\pi v_{t+1,LH-}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big) + (1-\pi)v_{t+1,LL-}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big)\Big]\Big\} \\
v_{t,HH+}(\hat{\pi}_{t-1}) &= \max\Big\{ \ln\Big(p\Big(\theta_H, (1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big)\Big) + \beta\Big[\pi v_{t+1,HH+}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big) + (1-\pi)v_{t+1,HL+}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big)\Big], \\
&\qquad \ln\Big(p\Big(\theta_L, (1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big)\Big) + \beta\Big[\pi v_{t+1,LH0}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big) + (1-\pi)v_{t+1,LL0}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big)\Big]\Big\} \\
v_{t,HL-}(\hat{\pi}_{t-1}) &= \max\Big\{ \ln\Big(p\Big(\theta_H, (1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big)\Big) + \beta\Big[(1-\pi)v_{t+1,HH0}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big) + \pi v_{t+1,HL0}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big)\Big], \\
&\qquad \ln\Big(p\Big(\theta_L, (1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big)\Big) + \beta\Big[(1-\pi)v_{t+1,LH-}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big) + \pi v_{t+1,LL-}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big)\Big]\Big\} \\
v_{t,HL0}(\hat{\pi}_{t-1}) &= \max\Big\{ \ln\Big(p\Big(\theta_H, (1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big)\Big) + \beta\Big[(1-\pi)v_{t+1,HH+}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big) + \pi v_{t+1,HL+}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big)\Big], \\
&\qquad \ln\Big(p\Big(\theta_H, (1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big)\Big) + \beta\Big[(1-\pi)v_{t+1,LH0}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big) + \pi v_{t+1,LL0}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big)\Big]\Big\} \\
v_{t,HL+}(\hat{\pi}_{t-1}) &= \ln\Big(p\Big(\theta_L, (1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big)\Big) + \beta\Big[(1-\pi)v_{t+1,LH+}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big) + \pi v_{t+1,LL+}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big)\Big] \\
v_{t,LH-}(\hat{\pi}_{t-1}) &= \ln\Big(p\Big(\theta_H, (1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big)\Big) + \beta\Big[\pi v_{t+1,HH-}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big) + (1-\pi)v_{t+1,HL-}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big)\Big] \\
v_{t,LH0}(\hat{\pi}_{t-1}) &= \max\Big\{ \ln\Big(p\Big(\theta_H, (1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big)\Big) + \beta\Big[\pi v_{t+1,HH0}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big) + (1-\pi)v_{t+1,HL0}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big)\Big], \\
&\qquad \ln\Big(p\Big(\theta_L, (1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big)\Big) + \beta\Big[\pi v_{t+1,LH-}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big) + (1-\pi)v_{t+1,LL-}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big)\Big]\Big\} \\
v_{t,LH+}(\hat{\pi}_{t-1}) &= \max\Big\{ \ln\Big(p\Big(\theta_H, (1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big)\Big) + \beta\Big[\pi v_{t+1,HH+}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big) + (1-\pi)v_{t+1,HL+}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big)\Big], \\
&\qquad \ln\Big(p\Big(\theta_L, (1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big)\Big) + \beta\Big[\pi v_{t+1,LH0}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big) + (1-\pi)v_{t+1,LL0}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big)\Big]\Big\} \\
v_{t,LL-}(\hat{\pi}_{t-1}) &= \max\Big\{ \ln\Big(p\Big(\theta_H, (1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big)\Big) + \beta\Big[(1-\pi)v_{t+1,HH0}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big) + \pi v_{t+1,HL0}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big)\Big], \\
&\qquad \ln\Big(p\Big(\theta_L, (1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big)\Big) + \beta\Big[(1-\pi)v_{t+1,LH-}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big) + \pi v_{t+1,LL-}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big)\Big]\Big\} \\
v_{t,LL0}(\hat{\pi}_{t-1}) &= \max\Big\{ \ln\Big(p\Big(\theta_H, (1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big)\Big) + \beta\Big[(1-\pi)v_{t+1,HH+}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big) + \pi v_{t+1,HL+}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}\Big)\Big], \\
&\qquad \ln\Big(p\Big(\theta_L, (1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big)\Big) + \beta\Big[(1-\pi)v_{t+1,LH0}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big) + \pi v_{t+1,LL0}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big)\Big]\Big\} \\
v_{t,LL+}(\hat{\pi}_{t-1}) &= \ln\Big(p\Big(\theta_L, (1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big)\Big) + \beta\Big[(1-\pi)v_{t+1,LH+}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big) + \pi v_{t+1,LL+}\Big((1-\tfrac{1}{t})\hat{\pi}_{t-1}+\tfrac{1}{t}\Big)\Big]
\end{aligned}
\tag{2.9.8}
$$

Expression 2. (Investors' probability assessment under the regime shifting beliefs.) If

$y_t = y_{t-1}$, investors' probability assessment of the current regime being mean-reverting is given by

$$q_t = q^1(q_{t-1}) \equiv \frac{[(1-\lambda_1)q_{t-1} + \lambda_2(1-q_{t-1})](1-\pi_H)}{[(1-\lambda_1)q_{t-1} + \lambda_2(1-q_{t-1})](1-\pi_H) + [\lambda_1 q_{t-1} + (1-\lambda_2)(1-q_{t-1})]\pi_H} \tag{2.9.9}$$

and if $y_t \neq y_{t-1}$,

$$q_t = q^2(q_{t-1}) \equiv \frac{[(1-\lambda_1)q_{t-1} + \lambda_2(1-q_{t-1})]\pi_H}{[(1-\lambda_1)q_{t-1} + \lambda_2(1-q_{t-1})]\pi_H + [\lambda_1 q_{t-1} + (1-\lambda_2)(1-q_{t-1})](1-\pi_H)} \tag{2.9.10}$$

Proof of Lemma 2.5. The price function can be rewritten as

$$p(y_t, q_t) = \sum_{s=1}^{\infty} \left(\frac{1}{1+\delta}\right)^s E_t[y_{t+s}|(y_t, q_t)]$$

I can decompose (perceived) earnings into a constant and a white noise,

$$y_t = \begin{cases} \theta_H = \frac{\theta_H+\theta_L}{2} + \frac{\theta_H-\theta_L}{2} \\ \theta_L = \frac{\theta_H+\theta_L}{2} - \frac{\theta_H-\theta_L}{2} \end{cases} \tag{2.9.11}$$

Therefore, $y_t = \frac{\theta_H+\theta_L}{2} + z_t$, where $z_t \in \{\frac{A}{2}, -\frac{A}{2}\}$ and $A = \theta_H - \theta_L$. It is apparent that investors would view z_t as following the same regime-shifting process as y_t. The price function can be written as

$$\begin{aligned} p(y_t, q_t) &= \sum_{s=1}^{\infty} \frac{1}{(1+\delta)^s} E_t\left[\frac{\theta_H+\theta_L}{2} + z_{t+s}|(z_t, q_t)\right] \\ &= \frac{\theta_H+\theta_L}{2\delta} + \sum_{s=1}^{\infty} \frac{1}{(1+\delta)^s} E_t[z_{t+s}|(z_t, q_t)] \end{aligned} \tag{2.9.12}$$

The next step revokes a result proved in Barberis et al. (1998), which is rephrased in

Lemma 2.6.

Lemma 2.6. (Barberis et al. 1998) If an investor believes that $z_t \in \{\frac{A}{2}, -\frac{A}{2}\}$ follows the regime-shifting process described above, her expectation is given by

$$E_t[z_{t+s}|(z_t, q_t)] = z_t\gamma_0' Q^s(\gamma_1 + \gamma_2 q_t) \tag{2.9.13}$$

where $\gamma_0 = [1, -1, 1, -1]'$, $\gamma_1 = [0, 0, 1, 0]'$, $\gamma_2 = [1, 0, -1, 0]'$, and

$$Q = \begin{bmatrix} (1-\lambda_1)(1-\pi_H) & (1-\lambda_1)\pi_H & \lambda_2(1-\pi_H) & \lambda_2\pi_H \\ (1-\lambda_1)\pi_H & (1-\lambda_1)(1-\pi_H) & \lambda_2\pi_H & \lambda_2(1-\pi_H) \\ \lambda_1\pi_H & \lambda_1(1-\pi_H) & (1-\lambda_2)\pi_H & (1-\lambda_2)(1-\pi_H) \\ \lambda_1(1-\pi_H) & \lambda_1\pi_H & (1-\lambda_2)(1-\pi_H) & (1-\lambda_2)\pi_H \end{bmatrix}$$

Now I proceed with the proof of Lemma 2.5. Based on (2.9.12), the price function can be written as

$$\begin{aligned} p(y_t, q_t) &= \frac{\theta_H + \theta_L}{2\delta} + z_t\gamma_0'\Big(\sum_{s=1}^{\infty}\frac{Q^s}{(1+\delta)^s}\Big)(\gamma_1 + \gamma_2 q_t) \\ &= \frac{\theta_H + \theta_L}{2\delta} + \frac{z_t}{1+\delta}\gamma_0'\big[I - \frac{Q}{1+\delta}\big]^{-1}Q(\gamma_1 + \gamma_2 q_t) \\ &= \frac{\theta_H + \theta_L}{2\delta} + \frac{y_t - (\theta_H + \theta_L)/2}{1+\delta}\gamma_0'\big[I - \frac{Q}{1+\delta}\big]^{-1}Q(\gamma_1 + \gamma_2 q_t) \end{aligned} \tag{2.9.14}$$

which completes the proof of Lemma 2.5.

Expression 3 (Proposition 2.4). The Bellman equations in Proposition 2.4 can be represented by 12 univariate functions, $v_{ijk}(q_{t-1})$, where $i, j \in \{H, L\}$ and $k \in \{-, 0, +\}$.

The functions the following relations, for $0 \le q_{t-1} \le 1$,

$$\begin{aligned}
v_{HH-}(q_{t-1}) &= \ln\Big(p(\theta_H, q^1(q_{t-1}))\Big) + \beta\Big[\pi v_{HH-}(q^1(q_{t-1})) + (1-\pi)v_{HL-}(q^1(q_{t-1}))\Big] \\
v_{HH0}(q_{t-1}) &= \max\Big\{\ln\Big(p(\theta_H, q^1(q_{t-1}))\Big) + \beta\Big[\pi v_{HH0}(q^1(q_{t-1})) + (1-\pi)v_{HL0}(q^1(q_{t-1}))\Big], \\
&\qquad \ln\Big(p(\theta_L, q^2(q_{t-1}))\Big) + \beta\Big[\pi v_{LH-}(q^2(q_{t-1})) + (1-\pi)v_{LL-}(q^2(q_{t-1}))\Big]\Big\} \\
v_{HH+}(q_{t-1}) &= \max\Big\{\ln\Big(p(\theta_H, q^1(q_{t-1}))\Big) + \beta\Big[\pi v_{HH+}(q^1(q_{t-1})) + (1-\pi)v_{HL+}(q^1(q_{t-1}))\Big], \\
&\qquad \ln\Big(p(\theta_L, q^2(q_{t-1}))\Big) + \beta\Big[\pi v_{LH0}(q^2(q_{t-1})) + (1-\pi)v_{LL0}(q^2(q_{t-1}))\Big]\Big\} \\
v_{HL-}(q_{t-1}) &= \max\Big\{\ln\Big(p(\theta_H, q^1(q_{t-1}))\Big) + \beta\Big[(1-\pi)v_{HH0}(q^1(q_{t-1})) + \pi v_{HL0}(q^1(q_{t-1}))\Big], \\
&\qquad \ln\Big(p(\theta_L, q^2(q_{t-1}))\Big) + \beta\Big[(1-\pi)v_{LH-}(q^2(q_{t-1})) + \pi v_{LL-}(q^2(q_{t-1}))\Big]\Big\} \\
v_{HL0}(q_{t-1}) &= \max\Big\{\ln\Big(p(\theta_H, q^1(q_{t-1}))\Big) + \beta\Big[(1-\pi)v_{HH+}(q^1(q_{t-1})) + \pi v_{HL+}(q^1(q_{t-1}))\Big], \\
&\qquad \ln\Big(p(\theta_L, q^2(q_{t-1}))\Big) + \beta\Big[(1-\pi)v_{LH0}(q^2(q_{t-1})) + \pi v_{LL0}(q^2(q_{t-1}))\Big]\Big\} \\
v_{HL+}(q_{t-1}) &= \ln\Big(p(\theta_L, q^2(q_{t-1}))\Big) + \beta\Big[(1-\pi)v_{LH+}(q^2(q_{t-1})) + \pi v_{LL+}(q^2(q_{t-1}))\Big] \\
v_{LH-}(q_{t-1}) &= \ln\Big(p(\theta_H, q^2(q_{t-1}))\Big) + \beta\Big[\pi v_{HH-}(q^2(q_{t-1})) + (1-\pi)v_{HL-}(q^2(q_{t-1}))\Big] \\
v_{LH0}(q_{t-1}) &= \max\Big\{\ln\Big(p(\theta_H, q^2(q_{t-1}))\Big) + \beta\Big[\pi v_{HH0}(q^2(q_{t-1})) + (1-\pi)v_{HL0}(q^2(q_{t-1}))\Big], \\
&\qquad \ln\Big(p(\theta_L, q^1(q_{t-1}))\Big) + \beta\Big[\pi v_{LH-}(q^1(q_{t-1})) + (1-\pi)v_{LL-}(q^1(q_{t-1}))\Big]\Big\} \\
v_{LH+}(q_{t-1}) &= \max\Big\{\ln\Big(p(\theta_H, q^2(q_{t-1}))\Big) + \beta\Big[\pi v_{HH+}(q^2(q_{t-1})) + (1-\pi)v_{HL+}(q^2(q_{t-1}))\Big], \\
&\qquad \ln\Big(p(\theta_L, q^1(q_{t-1}))\Big) + \beta\Big[\pi v_{LH0}(q^1(q_{t-1})) + (1-\pi)v_{LL0}(q^1(q_{t-1}))\Big]\Big\} \\
v_{LL-}(q_{t-1}) &= \max\Big\{\ln\Big(p(\theta_H, q^2(q_{t-1}))\Big) + \beta\Big[(1-\pi)v_{HH0}(q^2(q_{t-1})) + \pi v_{HL0}(q^2(q_{t-1}))\Big], \\
&\qquad \ln\Big(p(\theta_L, q^1(q_{t-1}))\Big) + \beta\Big[(1-\pi)v_{LH-}(q^1(q_{t-1})) + \pi v_{LL-}(q^1(q_{t-1}))\Big]\Big\} \\
v_{LL0}(q_{t-1}) &= \max\Big\{\ln\Big(p(\theta_H, q^2(q_{t-1}))\Big) + \beta\Big[(1-\pi)v_{HH+}(q^2(q_{t-1})) + \pi v_{HL+}(q^2(q_{t-1}))\Big], \\
&\qquad \ln\Big(p(\theta_L, q^1(q_{t-1}))\Big) + \beta\Big[(1-\pi)v_{LH0}(q^1(q_{t-1})) + \pi v_{LL0}(q^1(q_{t-1}))\Big]\Big\} \\
v_{LL+}(q_{t-1}) &= \ln\Big(p(\theta_L, q^1(q_{t-1}))\Big) + \beta\Big[(1-\pi)v_{LH+}(q^1(q_{t-1})) + \pi v_{LL+}(q^1(q_{t-1}))\Big]
\end{aligned} \tag{2.9.15}$$

2.9.2 An alternative setup with continuous earnings

This section sketches an alternative setup where reported earnings can be continuous and investors believe in an autoregressive earnings process.

Earnings process and financial reporting problem

Suppose real earnings $\theta_t \in \{\theta_H, \theta_L\} \equiv \Theta$ follow the same Markov process as the main model. (Assuming continuous values for real earnings complicates the analysis without gaining new insights.) However, the manager can now report continuous earnings, $y_t \in \mathbb{R}$, by manipulating accounting accruals $a_t = y_t - \theta_t$. The balance sheet constraint now requires that the accumulated accruals $x_t = x_{t-1} + a_t$ cannot exceed $\bar{x}$ in magnitude, i.e., $x_t \in [-\bar{x}, \bar{x}] \equiv X$.

As before, stock price is given by

$$p_t = \frac{1}{1+\delta} E_t[p_{t+1} + y_{t+1}] \tag{2.9.16}$$

where the expectation operator is defined by the specific expectations scheme discussed in next section.

The manager's objective function is the same as the main model,

$$\max_{y_t \in \{\theta_H, \theta_L\}} E_t\Big[\sum_{s=0}^{\infty} \beta^s \ln(p_{t+s})\Big] \tag{2.9.17}$$

Investor expectations

With continuous earnings, investor beliefs can no longer be represented by discrete probabilities. Take, for example, regime-shifting beliefs. To capture a perceived law of motion with switching regimes with different levels of persistence, I will need to use the discretization method proposed by Tauchen (1986) to obtain an Markov-chain approximation of a continuous AR(1) process. Beliefs in different AR(1) coefficients can be approximated by differential beliefs in the transition matrix of a Markov chain.[33] Specifi-

[33] Alternatively, I can define regime on other metrics applicable to continuous earnings, such as variance.

cally, based on Tauchen (1986), an AR(1) process, as the investor presumes, is equivalent to the following Markov matrix,

$$\pi \equiv \pi(y_{t+1}|y_t) = \begin{bmatrix} \pi_{11} & \cdots & \pi_{1N} \\ \vdots & \ddots & \vdots \\ \pi_{N1} & \cdots & \pi_{NN} \end{bmatrix}$$

where π_{jk} is given by

$$\pi_{jk} = \begin{cases} \Phi\Big(\bar{y}^1 - (1-\rho)\mu - \rho\bar{y}^j + \frac{\Delta}{2}\Big) & \text{for } k = 1 \text{ and } j = 1, ..., N \\ \Phi\Big(\bar{y}^k - (1-\rho)\mu - \rho\bar{y}^j + \frac{\Delta}{2}\Big) - \Phi\Big(\bar{y}^k - (1-\rho)\mu - \rho\bar{y}^j - \frac{\Delta}{2}\Big) & \text{for } 2 \le k \le N-1 \text{ and } j = 1, ..., N \\ 1 - \Phi\Big(\bar{y}^N - (1-\rho)\mu - \rho\bar{y}^j - \frac{\Delta}{2}\Big) & \text{for } k = N \text{ and } j = 1, ..., N \end{cases}$$

Another example is autoregressive beliefs: Investors perceive the law of motion for reported earnings as

$$y_t = (1-\rho)\mu + \rho y_{t-1} + \varepsilon_t \tag{2.9.18}$$

where μ is the prior of mean earnings, ρ is the autoregressive coefficient as perceived by investors, and ε_t is a white noise. This can be motivated by earnings forecasting models used by researchers, analysts, and other capital market participants (e.g., Brown 1993).

Therefore, after observing period t reported earnings, investor's expectation of period $t+s$ $(s = 1, 2, ...)$ earnings is

$$\begin{aligned} E_t[y_{t+s}|y_t] &= (1-\rho)\mu + \rho E_t[y_{t+s-1}|y_t] \\ &= (1-\rho)\mu + \rho\Big((1-\rho)\mu + \rho E_t[y_{t+s-2}|y_t]\Big) = ... \\ &= (1-\rho^s)\mu + \rho^s y_t \end{aligned} \tag{2.9.19}$$

and it is easy to show that the pricing function is given by

$$p(y_t) = \frac{\mu}{\delta} + \frac{\rho}{1+\delta-\rho}(y_t - \mu) \tag{2.9.20}$$

In this case, the manager's optimization problem can be described by the following Bellman equations,

$$v_H(x_{t-1}) = \max_{a_t \in \Gamma(x_{t-1})} \left\{ \ln\Big(p(\theta_H + a_t)\Big) + \beta\Big(\pi v_H(x_{t-1}+a_t) + (1-\pi) v_L(x_{t-1}+a_t)\Big)\right\} \tag{2.9.21}$$

$$v_L(x_{t-1}) = \max_{a_t \in \Gamma(x_{t-1})} \left\{ \ln\Big(p(\theta_L + a_t)\Big) + \beta\Big((1-\pi) v_H(x_{t-1}+a_t) + \pi v_L(x_{t-1}+a_t)\Big)\right\} \tag{2.9.22}$$

where $\Gamma(x_{t-1}) = [-\bar{x} - x_{t-1}, \bar{x} - x_{t-1}]$ is the support for the decision rule a_t conditional on last period's accumulated accruals. Similarly, this problem can be solved by value function iteration.

Stationarity and equilibrium

Equilibrium concepts are defined similarly to the discrete case. In the main model, the particularly simple form of condition for stationarity is made possible by the discreteness and small number of realized values of state variables. With continuous earnings, however, defining the stationarity of an equilibrium inevitably involves applications of measure theory.

A stationary rational expectations equilibrium is a set of decision rules $y(\theta_t, x_{t-1})$, price function $p(y_t)$, and a probability measure ψ such that

(i) Reporting decision solves the manager's optimization problem;

(ii) Investor expectations are consistent with ψ and $y(\cdot)$;

(iii) The probability measure ψ is a stationary, i.e.,

$$\psi(B) = \int_S P(s_t, B) d\psi \tag{2.9.23}$$

where ψ is a stationary probability measure on (S, β_S), where $S = \Theta \times X$ is the state space, β_S is the Borel σ-algebra on S, $B \in \beta_S$, $s_t = (\theta_t, x_{t-1})$ is the state vector, $P(s_t, B)$ is the probability that this period state variable s_t leads to $s_{t+1} \in B$ next period.

2.10 References

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Table 2.1: Underreaction and Overreaction to Earnings Reports

This table reports the returns to portfolios formed based on the previous n periods of earnings, under three investor expectations schemes. For each scheme, I first simulate 1000 idiosyncratic firms for 50 periods. For each period with sufficient data, I form a zero-investment portfolio by buying all firms with positive earnings in each of the previous n periods, and shorting all firms with negative earnings in each of the n periods. I use the equal-weighted return as portfolio return, and report the average return to the portfolios formed on all n periods. Returns are in percent.

Expectations Scheme	Adaptive Learning		Regime-shifting Beliefs	
Consecutive periods (n)	Unmanaged	Managed	Unmanaged	Managed
1	-6.448	-5.637	1.038	0.869
2	-6.718	-5.896	0.466	0.381
3	-7.113	-6.212	0.003	0.032
4	-7.589	-6.576	-0.263	-0.246
5	-8.042	-6.987	-0.387	-0.351

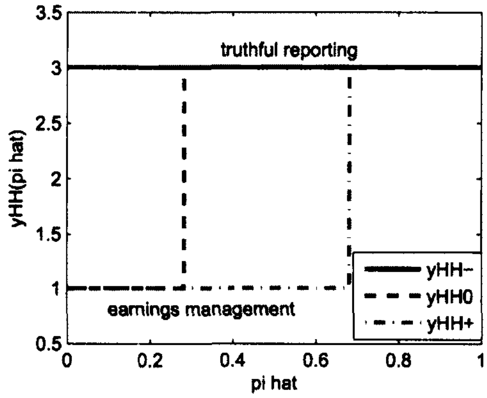
Table 2.2: The Accruals Anomaly

This table reports the performance of accruals-based portfolios over 5 periods, under three investor expectations schemes. For each scheme, I first simulate 1000 idiosyncratic firms for 50 periods. For each period with sufficient data, I form a zero-investment portfolio by buying all firms with negative accruals ($a_t = -A$) and shorting all firms with positive accruals ($a_t = A$). In subsequent 5 periods, I use the equal-weighted returns as portfolio return, and report the average return to all portfolios. Returns are in percent.

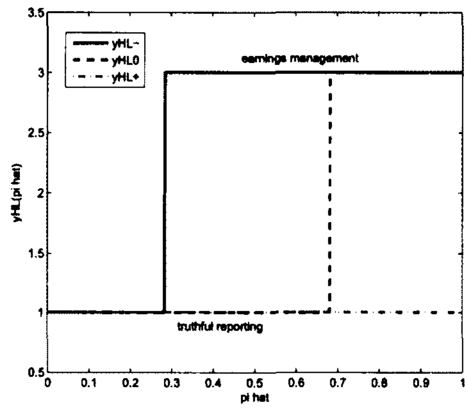
Expectations Scheme	Adaptive Learning			Regime-shifting Beliefs		
Future period	$a = -A$	$a = A$	Accruals Anomaly	$a = -A$	$a = A$	Accruals Anomaly
1	2.110	-2.628	4.802	-0.451	0.402	-0.854
2	2.883	-0.733	3.894	0.077	-0.271	0.349
3	1.035	-0.900	1.153	0.249	-0.208	0.457
4	0.316	-0.369	0.282	0.009	0.124	-0.115
5	-0.565	0.408	-0.687	0.035	-0.030	0.065

Figure 2.1: Decision Rules under Adaptive Learning

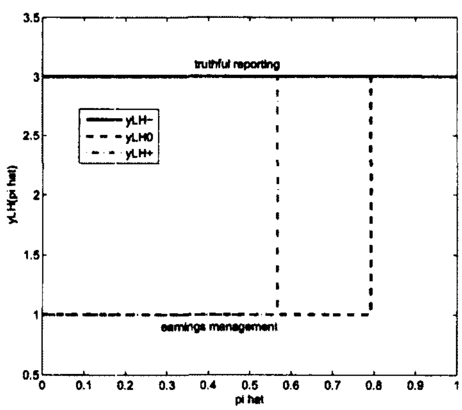
Decision rules $y_{ijk}(q_{t-1})$ are 12 univariate functions of q_{t-1}, where the subscripts i, j and k correspond to $y_{t-1} \in \{\theta_H, \theta_L\}$, $\theta_t \in \{\theta_H, \theta_L\}$, and $x_{t-1} \in \{-A, 0, A\}$, respectively. Panels (a)-(d) report the decision rules for period 3. Panels (e)-(f) report the variations in misreporting thresholds over the first 20 periods, for an alternative real earnings process with $\pi = 0.75$.



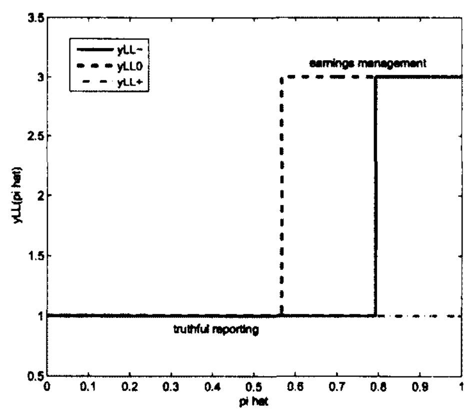
(a) $y_{HH}(\hat{\pi}_{t-1})$



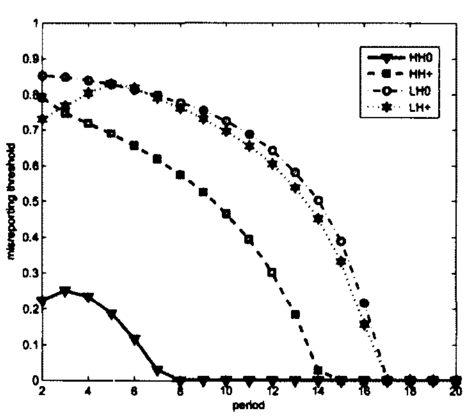
(b) $y_{HL}(\hat{\pi}_{t-1})$



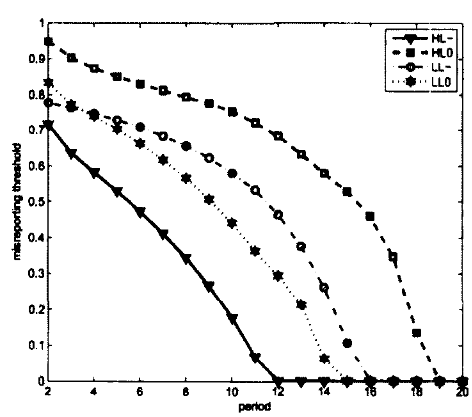
(c) $y_{LH}(\hat{\pi}_{t-1})$



(d) $y_{LL}(\hat{\pi}_{t-1})$



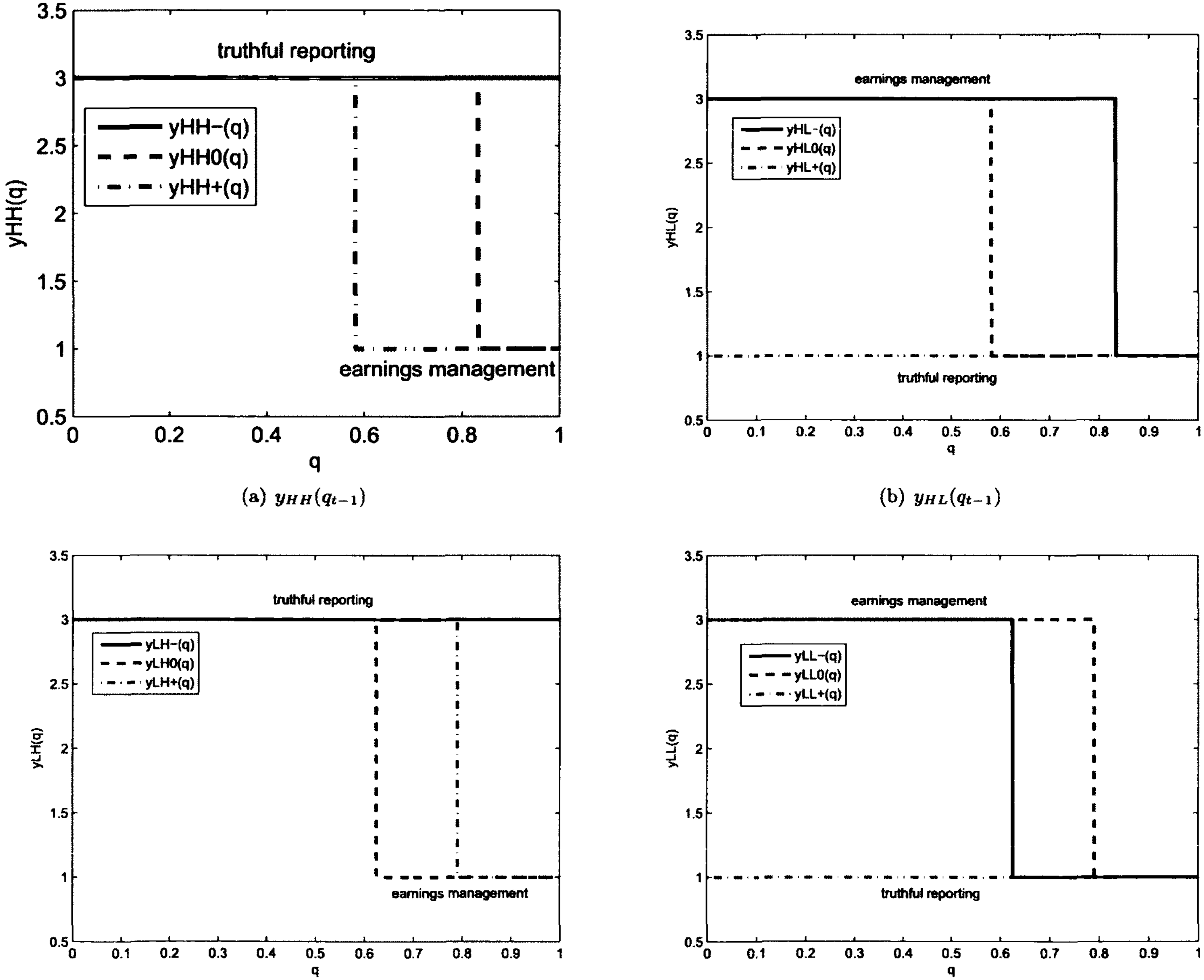
(e) Misreporting thresholds ($\theta_t = \theta_H$)



(f) Misreporting thresholds ($\theta_t = \theta_L$)

Figure 2.2: Decision Rules under Regime-shifting Belief

Decision rules $y_{ijk}(q_{t-1})$ are 12 univariate functions of q_{t-1}, where the subscripts i, j and k correspond to $y_{t-1} \in \{\theta_H, \theta_L\}$, $\theta_t \in \{\theta_H, \theta_L\}$, and $x_{t-1} \in \{-A, 0, A\}$, respectively.



(a) $y_{HH}(q_{t-1})$

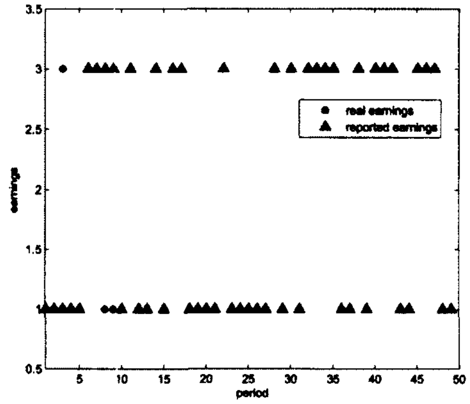
(b) $y_{HL}(q_{t-1})$

(c) $y_{LH}(q_{t-1})$

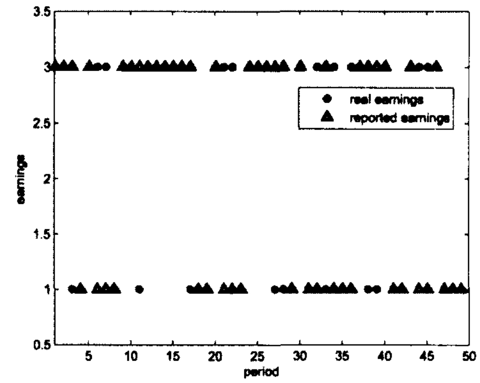
(d) $y_{LL}(q_{t-1})$

Figure 2.3: Earnings, Investor Beliefs, and Prices under Adaptive Learning and Regime-shifting Belief

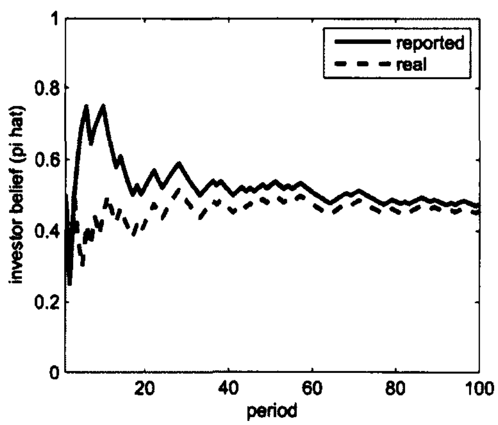
This figure compares the histories of reported and real earnings, investor beliefs, and stock prices under adaptive learning and regime-shifting schemes. For beliefs and prices, I also calculate their hypothetical counterparts based on real earnings as if they were observed by investors (dashed line 'real').



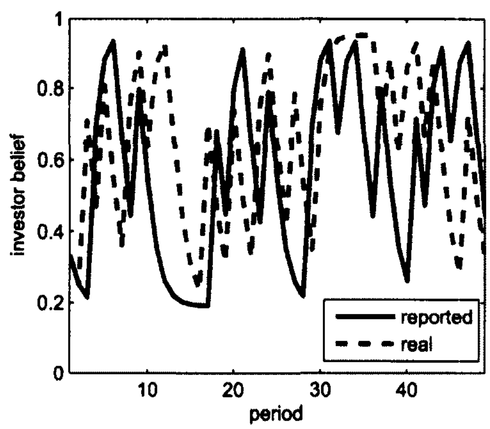
(a) Reported earnings under adaptive learning



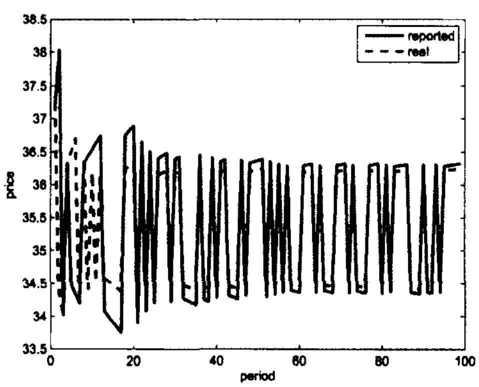
(b) Reported earnings under regime-shifting



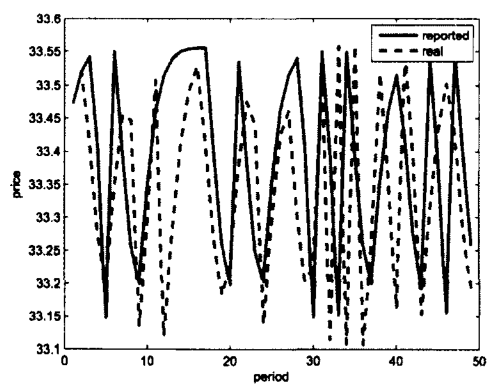
(c) Investor beliefs ($\hat{\pi}_t$) under adaptive learning



(d) Investor beliefs (q_t) under regime-shifting



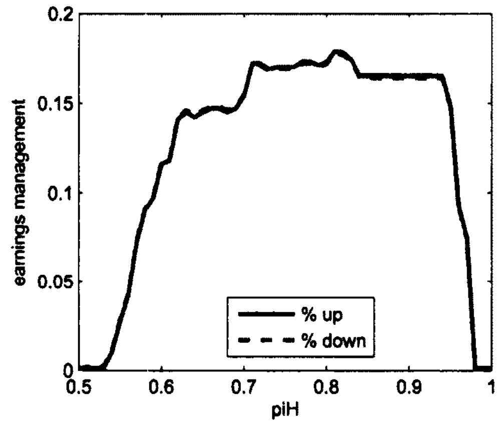
(e) Price under adaptive learning



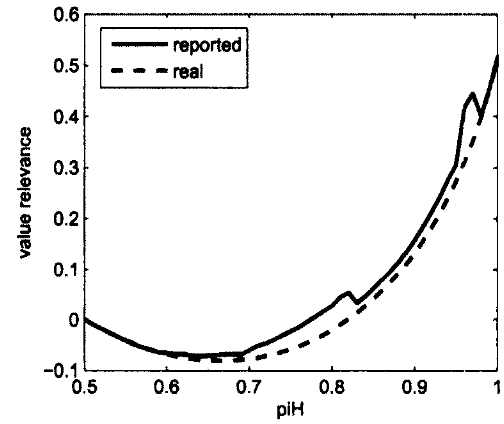
(f) Price under regime-shifting

Figure 2.4: The Role of Regime Heterogeneity under Regime-shifting Belief

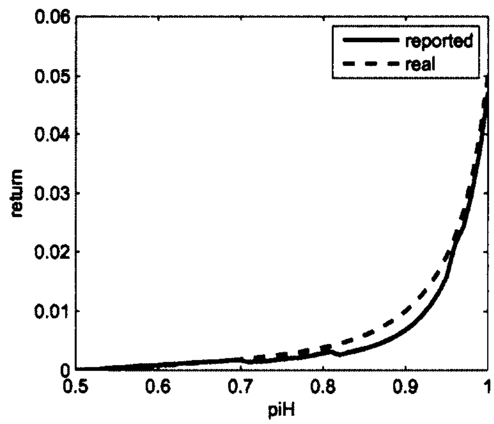
The figure reports the comparative statics of six earnings and return metrics with respect to the regime heterogeneity as perceived by investors under the regime-shifting scheme.



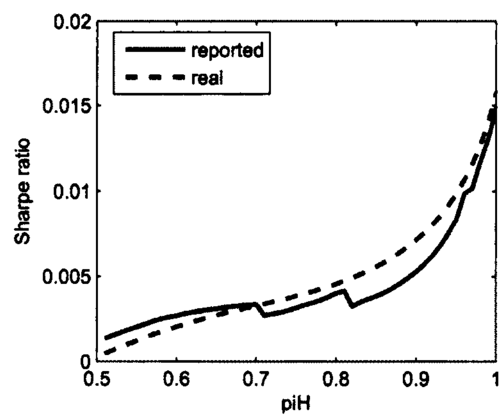
(a) Prevalence of earning management



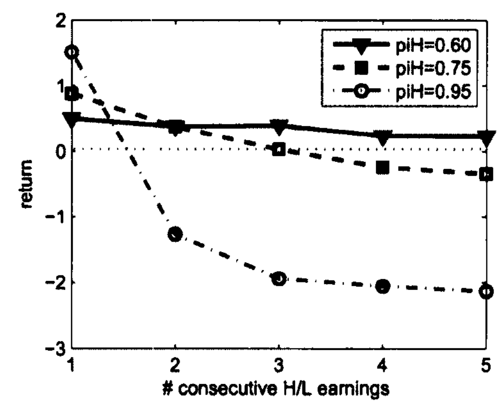
(b) Value relevance



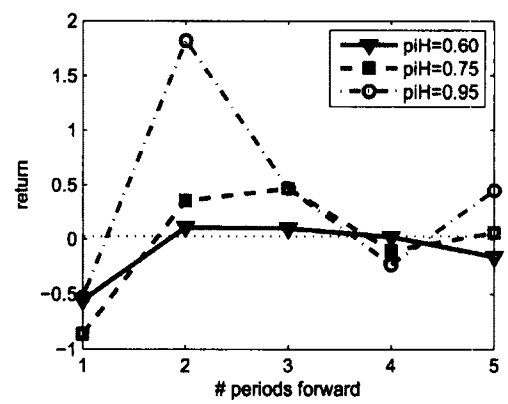
(c) Mean return



(d) Sharpe ratio



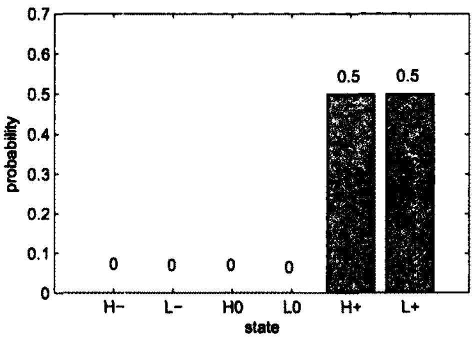
(e) Underreaction and overreaction



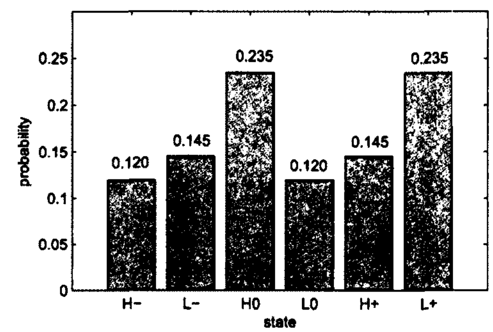
(f) The accruals anomaly

Figure 2.5: Stationary Distribution and Return Regularities under Adaptive Learning and Regime-shifting Belief

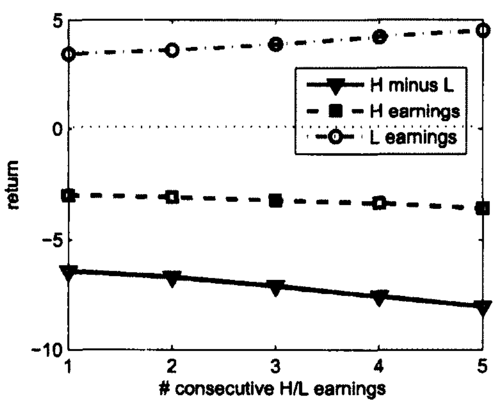
This figure compares the stationary distribution of state variables (θ_t, x_{t-1}), underreaction and overreaction to earnings, and the accruals anomaly, under adaptive learning and regime-shifting schemes.



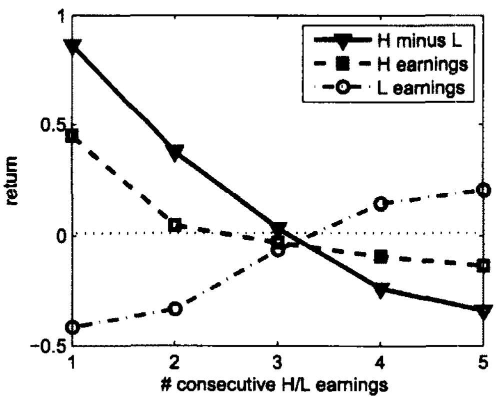
(a) Distribution under adaptive learning



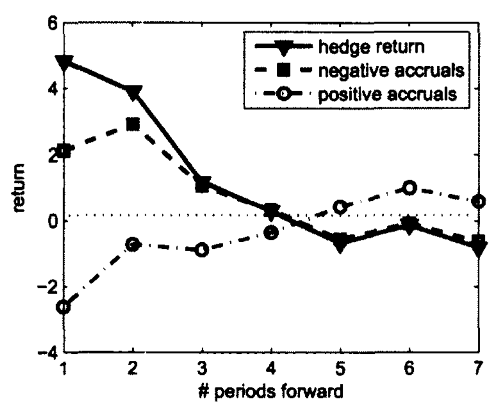
(b) Distribution under regime-shifting



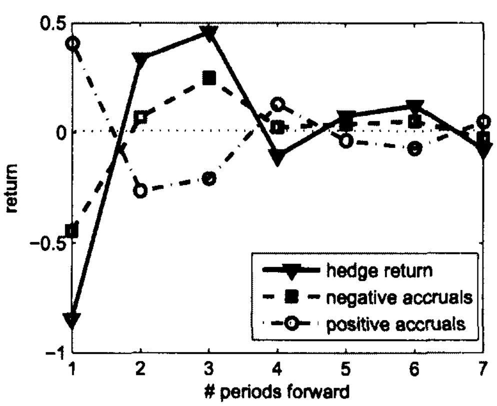
(c) Under-/overreaction under adaptive learning



(d) Under-/overreaction under regime-shifting



(e) Accruals anomaly under adaptive learning



(f) Accruals anomaly under regime-shifting

Chapter 3

Information Aggregation and Multiplicity in Creditor Runs: The Role of Public Disclosure

3.1 Introduction

Creditors of a distressed firm may trade equity shares or credit derivatives of the same firm they finance. The price of the financial asset is a signal that aggregates dispersed private information among creditors, and is incorporated into creditors trading and lending decisions. When a large portion of creditors decide not to rollover their lending, the firm could be forced to file bankruptcy. In this paper, I study the role of public disclosure in a two-period model in which creditors first trade a financial asset of the firm, and then coordinate on whether to continue to lend to the firm. In particular, I investigate how the quality of public disclosure influences the equilibrium outcome of creditors' coordination game.

In recent years, financial institutions increasingly trade public securities of the same companies to which they provide financing. Traded securities include equity shares, credit derivatives, and other financial contracts whose reference entities are their debtors in the lending relationship. As of 2006, over 70% of high-yield loans including leveraged buyouts and mergers and acquisitions financing are held by institutional investors (Ivashina and Sun 2010). Based on survey data from the Bank for International Settlements (BIS),[1] the total notional amount of the credit default swap (CDS) market was $57 trillion by June 2008, though it was reduced to $30 trillion by the end of 2010. Figure 3.1 as reproduced from Duffie (2008) shows the trend of banks' total CDS holding compared with total loans. Apparently, the large position of CDS held by financial institutions relative to their loan sizes indicates that banks hold CDS not only for hedging purpose, but also for speculative trading. Research shows that institutional investors use private information acquired from the loan market in trading public securities (e.g., Acharya and Johnson 2007; Ivashina and Sun 2010).

When creditors trade financial assets of the borrowing firm, their private information is incorporated into asset prices. Empirical studies show that prices of various assets, such as equity shares and credit derivatives, contain information that is not otherwise available. For example, it is well documented that stock prices may guide corporate decisions by conveying aggregated information of equity traders (e.g., Luo 2005; Chen, Goldstein, and Jiang 2007). It has also been shown that CDS price is a informative signal of the probability of corporate default, and the informativeness of CDS price is not confounded by contractual provisions of a bond, such as covenants, coupon, and maturity. As a result, information mostly flows from CDS price to bond price (Blanco et al. 2005). Due to the existence of counterparty risk, CDS price contains information about the probability of joint default of both the reference entity and "insurer" (Giglio 2011). In addition, the information

[1] Available at http://www.bis.org/statistics/derstats.htm.

contained in CDS price may be more important or preferable than credit ratings by rating agencies such as Standard & Poor's (S&P) and Moody's, as rating agencies face serious conflicts of interest (e.g., Partnoy 2006).

The informational role of price is especially important for creditors' decisions, because creditors have strong incentives to coordinate their moves based on all available public information. The strategic complementarity among creditors comes from the fact that corporate default is not solely based on the fundamental of the firm, but also on their beliefs of how others are likely to act. Even if a firm is relatively healthy and robust, a coordinated withdrawal of funding can still force the firm into bankruptcy. Therefore, a creditor is reluctant to provide additional financing if she believes that other creditors will not rollover their lending. For example, after the failure of Lehman Brothers in September 2008, there was a run by short-term bank creditors, making it difficult for banks to roll over their short term debt (Ivashina and Scharfstein 2010), and consequently a lot of banks went bankrupt. Exploiting a natural experiment that forced lenders to share negative private assessments about their borrowers, Hertzberg, Liberti, and Paravisini (2011) find that lenders to the same financially distressed firm have incentives to coordinate, and that public information exacerbates lender coordination and increases the incidence of distress or default. There is also abundant anecdotal evidence that creditors are proactively involved in bankruptcy negotiations.[2]

The informational role of creditor trading has profound implications for the practice of corporate disclosure, especially for distressed firms. The trading of financial assets aggregates dispersed, idiosyncratic information among creditors. Without creditor trading, public disclosure by the company is the dominant source of public information. With

[2]See, for examples, (i) "Hedge-fund Lending to Distressed Firms Makes for Gray Rules and Rough Play." *Wall Street Journal*, July 18, 2005; (ii) "CDS Derivatives Are Blamed for Role in Bankruptcy Filings." *Financial Times*, April 17, 2009; (iii) "CDS Investors Hold the Cards." *Financial Times*, July 22, 2009; (iv) "Credit Insurance Hampers GM Restructuring." *Financial Times*, May 11, 2009.

creditor trading, however, creditors can now observe another informative signal, asset price, which can potentially coordinate their actions. How public disclosure interacts with private information and the price signal in a coordination game remains an unexplored question.

In this paper, I examine the role of public disclosure in a coordination game in which creditors trade a financial asset of the firm they finance. Creditors learn from three sources of information, private information, public disclosure, and asset prices. In the first period of the two-period game, creditors trade the financial asset; and in the second period, they decide whether to rollover lending to the firm.

I show that the quality of public disclosure that the firm commits to plays a key role in determining equilibrium outcome of corporate default, through its interaction with private information. Specifically, when public disclosure is sufficiently noisy, unique equilibrium only obtains for intermediate values of the precision of private information; when public disclosure is sufficiently precise, multiple equilibria always emerge.

I then study the comparative statics of ex ante run probability and creditor welfare with respect to the precision of public information. I find that the welfare implications of public disclosure critically depends on the quality of private information and, when multiple equilibria exist, which equilibrium is selected. I also show that the results hold for two categories of general payoff structures for financial assets, which correspond to equity shares and credit derivatives.

This paper is related to studies on the role of public information in coordination games (e.g., Morris and Shin 2002; Svensson 2006; Angeletos et al. 2006), micro-founded macroeconomies (e.g., Amador and Weill 2010), and business-cycle models (e.g., Angeletos, Iovino, and La'O 2011). More closely, this paper is related to the global games literature including Morris and Shin (1998; 2000; 2004), Hellwig (2002), Angeletos and Werning (2006), and

Hellwig, Mukherji, and Tsyvinski (2006). In particular, Angeletos and Werning (2006) study an information aggregation model where traders may trade a derivative asset prior to the currency crises game. Hellwig, Mukherji, and Tsyvinski (2006) study a currency attack game in which interest rates are determined by the market. My paper contributes to this line of studies by examining the role of public disclosure in determining uniqueness versus multiplicity of the coordination game. A more detailed discussion on the connection between my results and findings of prior studies is deferred to Section 3.5.

By examining the information aggregation role of prices, this study is also related to the information aggregation literature (e.g., Hellwig 1980; Diamond and Verrecchia 1981; Plott and Sunder 1982), and studies on feedback effects that arise from the informational role of prices (e.g., Ozdenoren and Yuan 2008; Goldstein and Guembel 2008; Albagli, Hellwig, and Tsyvinski 2011; Goldstein, Ozdenoren, and Yuan 2011).

The institutional setting of this paper—a coordination game by creditors who also trade a financial asset of the borrowing firm—has also been examined empirically, with emphasis on insider trading by creditors (e.g., Acharya and Johnson 2007; Ivashina and Sun 2010). For example, Acharya and Johnson (2007) find evidence of trading based on non-public information by informed banks. Other theoretical studies that examine this setting include Bolton and Oehmke (forthcoming) and Zachariadis and Olaru (2010), but their focuses are distinct from this paper. In addition, Qu (2012) conducts experiments to study the role of public information in coordination games.

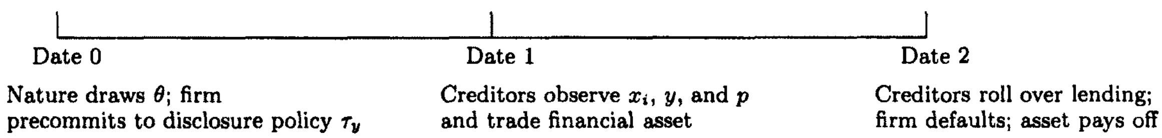
The rest of the paper is organized as follows. In Section 3.2, I introduce the main model in which creditors trade equity shares of the borrowing firm. I solve for the equilibrium and discuss the role of public disclosure on equilibrium multiplicity. In Section 3.3, I study the impact of public disclosure on expected run probability and creditor welfare. In Section 3.4, I study an alternative setup in which creditors trade credit derivatives of the firm. In

Section 3.5 I discuss the implications of the findings in the context of related studies. In Section 3.6 I conclude.

3.2 The Model

3.2.1 Trading and voting by creditors

I introduce commitment to public disclosure to Angeletos and Werning's (2006) model of coordination game with financial asset trading. This model is also related to Morris and Shin (1998; 2000; 2004), and Hellwig, Mukherji, and Tsyvinski (2006), among others. There is a continuum of creditors indexed by $i \in [0, 1]$ who lend to a firm (the debtor) and also trade the firm's shares or other derivative financial assets. The trading takes place prior to a coordination game which I call "creditors' run". The timeline of the model is summarized as below. The model timeline resembles the actual timeline of bankruptcy as documented in Ivashina et al. (2012), which is reproduced in Figure 3.2.



Timeline of the model

Information. At date 0, nature draws θ, the fundamental of the firm, from an improper uniform distribution over the entire real line.[3] The firm precommits to a disclosure policy which stipulates the precision of the firm's public disclosure at date 1 about θ, $y = \theta + \sigma_y \eta$, where $\sigma_y > 0$ and $\eta \sim N(0, 1)$ is i.i.d. In other words, the disclosure policy is set before the underlying fundamental is realized and observed by the firm. Precommitted disclosure policy is also seen in Gao (2010), who studies the role of public disclosure on ex ante cost of

[3]The assumption of improper prior is for expositional simplicity only. It reduces the number of parameters in the Bayesian updating of beliefs and is inconsequential when (i) signals about θ are sufficiently more precise than the prior distribution of θ and (ii) this prior has a continuous density.

capital and investor welfare. This perspective is different from models in which signals are strategically set by the informed party after the fundamental is realized (e.g., Angeletos, Hellwig, and Pavan 2006).

Financial market. At date 1, the firm makes a public disclosure y, and creditors each receives a noisy signal about θ, $x_i = \theta + \sigma_x \xi_i$, where $\xi_i \sim N(0,1)$ is i.i.d. I denote the precisions of the private information as $\tau_x \equiv \sigma_x^{-2}$, and the precision of the public disclosure by $\tau_y \equiv \sigma_y^{-2}$. Based on the public disclosure y and their private signals x_i, creditors trade a financial asset whose payoff is related to the firm's fundamental, θ. In the baseline model, I assume that the financial asset represents equity shares with payoff θ. In Section 3.4 I will study an alternative financial asset, credit derivative, whose payoff also depends on the aggregate size of attack in the second stage. Creditors have a CARA utility function, $v(w_i) = -e^{\gamma w_i}$ for $\gamma > 0$, where $w_i = w_0 - pk_i + \theta k_i$ is the wealth, w_0 is the initial wealth, and k_i is the demand for the financial asset by creditor i. The supply of the asset is given by $S = \sigma_\varepsilon \varepsilon$, where $\sigma_\varepsilon > 0$ and $\varepsilon \sim N(0,1)$ and i.i.d. Let $\tau_\varepsilon \equiv \sigma_\varepsilon^{-2}$. A market-clearing price sets aggregate demand to aggregate supply of the financial asset.

Creditor runs. At date 2, creditors decide whether to rollover their lending to the firm. Their rollover decision is denoted by a_i,

$$a_i = \begin{cases} 1 & \text{Terminate lending} \\ 0 & \text{Rollover lending} \end{cases} \tag{3.2.1}$$

The aggregate fraction of creditors who decide not to rollover their lending, is given by $A \equiv \int_i a_i di$. If a sufficient large fraction of creditors attack, i.e., $A > \theta$, the firm will be forced to file bankruptcy. If a creditor does not rollover, her payoff is 0 regardless of incidence of default; if a creditor rollovers her lending, the payoff is $1 - c$ if the firm remains viable (no default), and $-c$ if the firm goes bankrupt (default), where c can be interpreted as creditors' cost of providing financing to the firm. To summarize, creditors' payoff matrix for the debt-rollover game is as follows,

Payoff	No default	Default
Terminate lending	0	0
Rollover lending	$1-c$	$-c$

3.2.2 Equilibrium

I define an equilibrium in the two-stage game as follows.

Definition 3.1. An equilibrium is a set of price function $p(\theta,\eta,\varepsilon)$, creditor's demand for the financial asset $k_i(x_i,y,p)$, and creditor's lending decision $a_i(x_i,y,p)$, such that

(i) At date 1, creditors set their demands for the financial asset to maximize their utility from this period,

$$k_i(x_i,y,p) \in \text{argmax}_{k_i}\ E\big[v(w_0 - pk_i + \theta k_i)|(x_i,y,p)\big] \tag{3.2.2}$$

(ii) Price $p(\theta,\eta,\varepsilon)$ clears the market, i.e.,

$$K(\theta,y,p) = \sigma_\varepsilon \varepsilon \tag{3.2.3}$$

where aggregate demand function $K(\cdot)$ is given by

$$K(\theta,y,p) = \int_{i\in[0,1]} k_i(x_i,y,p)di \tag{3.2.4}$$

(iii) At date 2, creditors decide whether to rollover their lending to maximize expected payoff from this stage,

$$a_i(x_i, y, p) \in \text{argmax}_{a_i \in 0,1} \ (1 - a_i)E\big[\mathbf{1}_{A>\theta} - c|(x_i, y, p)\big] \tag{3.2.5}$$

(iv) The firm defaults if the fraction of creditors who attack (i.e., do not rollover their lending) is larger than θ,

$$A(\theta, y, p) = \int_{i \in [0,1]} a_i(x_i, y, p) di > \theta \tag{3.2.6}$$

3.2.3 Solution

I first solve for the equilibrium of the financial asset market. The equilibrium price of the asset aggregates creditors' private information, and is given by the following lemma.

Lemma 3.1. The price of the financial asset is given by

$$p(\theta, \eta, \varepsilon) = \theta + \frac{\tau_y \eta - \gamma \sigma_\varepsilon \varepsilon}{\tau_x + \tau_y} \tag{3.2.7}$$

and the precision of p as a signal of θ is given by

$$\tau_p \equiv \sigma_p^{-2} = \frac{(\tau_x + \tau_y)^2}{\tau_y^2 + \gamma^2/\tau_\varepsilon} \tag{3.2.8}$$

Proof. From (3.2.2), the demand function of creditor i is

$$k_i(x_i, y, p) = \frac{E[\theta|(x_i, y, p)] - p}{\gamma \text{var}[\theta|(x_i, y, p)]} \tag{3.2.9}$$

Suppose the price is a noisy signal of θ with variance σ_p^2 and precision $\tau_p = \sigma_p^{-2}$. Conditional on observing x_i, y, and p, creditor i's expectation of θ is given by $E[\theta|(x_i, y, p)] =$

$\frac{\tau_x x_i+\tau_y y+\tau_p p}{\tau_x+\tau_y+\tau_p}$ and $\text{var}[\theta|(x_i,y,p)] = \frac{1}{\tau_x+\tau_y+\tau_p}$.

Therefore, the demand of creditor i is given by

$$k_i(x_i,y,p) = \frac{1}{\gamma}\big(\tau_x(x_i-p)+\tau_y(y-p)\big) \tag{3.2.10}$$

By law of large numbers, $\int_i x_i di = \theta$, so the aggregate demand is

$$K(\theta,y,p) = \frac{1}{\gamma}\big(\tau_x(\theta-p)+\tau_y(y-p)\big) \tag{3.2.11}$$

The price function follows immediately from market clearing condition (3.2.3). ☐

At date 2, creditors decide whether to attack the firm by withdrawing lending. I focus on a linear monotone equilibrium with threshold rule: Creditors attack the firm whenever $x_i \leq x^*(y,p)$ or equivalently $\xi_i \leq \sqrt{\tau_x}(x^*(y,p)-\theta)$, and do not attack the firm when $x_i > x^*(y,p)$, i.e.,

$$a_i = \begin{cases} 1 & \text{if } x_i \leq x^*(y,p) \\ 0 & \text{if } x_i > x^*(y,p) \end{cases} \tag{3.2.12}$$

The decision threshold and default threshold are given by the following proposition.

Proposition 3.1. In a linear monotone equilibrium, the decisions threshold $x^*(y,p)$ and default threshold $\theta^*(y,p)$ are determined by the following two equations,

$$x^*(y,p) = \theta^* + \frac{1}{\sqrt{\tau_x}}\Phi^{-1}(\theta^*) \tag{3.2.13}$$

$$F(\theta^*,y,p) \equiv \frac{1}{\sqrt{\tau_x}}\Big(\tau_y y+\tau_p p-(\tau_y+\tau_p)\theta^*\Big)+\Phi^{-1}(\theta^*)-\sqrt{1+\frac{\tau_y+\tau_p}{\tau_x}}\Phi^{-1}(c) = 0 \tag{3.2.14}$$

where $\tau_p = \frac{(\tau_x+\tau_y)^2}{\tau_y^2+\gamma^2/\tau_\epsilon}$.

Proof. The equilibrium thresholds are characterized by two equations. The aggregate size of the attack is equal to the fraction of creditors who choose to attack, or the fraction of creditors whose private signals are below the threshold level,

$$A(\theta, y, p) = \Phi\big(\sqrt{\tau_x}(x^*(y,p) - \theta)\big) \tag{3.2.15}$$

where $\Phi(\cdot)$ is the c.d.f. of standard normal distribution. Let the default threshold $\theta^*(y,p)$ be the solution to $A(\theta, y, p) = \theta$, or

$$\Phi\big(\sqrt{\tau_x}(x^*(y,p) - \theta^*)\big) = \theta^* \tag{3.2.16}$$

which leads to the first equation of x^* and θ^*,

$$x^*(y,p) = \theta^* + \frac{1}{\sqrt{\tau_x}}\Phi^{-1}(\theta^*) \tag{3.2.17}$$

The other equation is based on marginal creditor's indifference condition. Creditors whose private signals are at the threshold level $x_i = x^*(y,p)$ are indifferent between the two decisions, attack or not attack, i.e., their expected payoffs from the two decisions are the same based on $x_i = x^*(y,p)$,

$$\Pr(\theta \leq \theta^*(y,p) | (x^*(y,p), y, p)) = 1 - c \tag{3.2.18}$$

Recall that conditional on (x_i, y, p), creditors' expectation of θ is $E[\theta|(x_i, y, p)] = \frac{\tau_x x_i + \tau_y y + \tau_p p}{\tau_x + \tau_y + \tau_p}$ and conditional variance is $\text{var}[\theta|(x_i, y, p)] = \frac{1}{\tau_x + \tau_y + \tau_p}$, where $\tau_p = \frac{(\tau_x + \tau_y)^2}{\tau_y^2 + \gamma^2/\tau_\epsilon}$. Equation

(3.2.18) becomes

$$\Phi\Big(\sqrt{\tau_x+\tau_y+\tau_p}\Big(\theta^*(y,p)-\frac{\tau_x x^*(y,p)+\tau_y y+\tau_p p}{\tau_x+\tau_y+\tau_p}\Big)\Big)=1-c \tag{3.2.19}$$

which is equivalent to

$$\theta^*(y,p)-\frac{\tau_x x^*(y,p)+\tau_y y+\tau_p p}{\tau_x+\tau_y+\tau_p}=\frac{1}{\sqrt{\tau_x+\tau_y+\tau_p}}\Phi^{-1}(1-c) \tag{3.2.20}$$

Combining (3.2.17) and (3.2.20), we can eliminate $x^*(y,p)$ and obtain a nonlinear equation of θ^* given by (3.2.14). □

3.2.4 Public disclosure and multiplicity

The following proposition characterizes the role of public disclosure in determining multiplicity versus uniqueness of the equilibrium in the coordination game played by creditors.

Proposition 3.2. (Determinacy of equilibrium) (i) If $\frac{\tau_y+\tau_p}{\sqrt{\tau_x}} \le \sqrt{2\pi}$, or if $\frac{\tau_y+\tau_p}{\sqrt{\tau_x}} > \sqrt{2\pi}$ and $s \equiv \tau_y y + \tau_p p \notin (\underline{s}, \bar{s})$, then there exists a unique equilibrium;

(ii) If $\frac{\tau_y+\tau_p}{\sqrt{\tau_x}} > \sqrt{2\pi}$ and $s \in (\underline{s}, \bar{s})$, where

$$\bar{s}=(\tau_y+\tau_p)\bar{\theta}-\sqrt{\tau_x}\Phi^{-1}(\bar{\theta})+\sqrt{\tau_x+\tau_y+\tau_p}\Phi^{-1}(c)$$

$$\underline{s}=(\tau_y+\tau_p)\underline{\theta}-\sqrt{\tau_x}\Phi^{-1}(\underline{\theta})+\sqrt{\tau_x+\tau_y+\tau_p}\Phi^{-1}(c)$$

and $\bar{\theta}=\Phi\left(\left|\phi^{-1}\left(\frac{\sqrt{\tau_x}}{\tau_y+\tau_p}\right)\right|\right)$ and $\underline{\theta}=1-\bar{\theta}$, then there exist multiple equilibria.

Intuitively, the determinacy of equilibrium depends on the ratio of the standard deviation of private signals to the variance of public signals. This result is in line with the global games literature (e.g., Morris and Shin 2000; Hellwig 2002). In my model, $\tau_y + \tau_p$ is the total precision of two public signals: the public disclosure y and the price p which aggregates creditors' private information. If I replace $\tau_y + \tau_p$ by the precision of a single, exogenous source of public information, this result follows directly from the main finding of Morris and Shin (2000), who also provide heuristic derivations.

I then study how noise in aggregate supply influences equilibrium. In Angeletos and Werning (2006), there is no exogenous public disclosure, and the condition for obtaining multiplicity is $\frac{\tau_x^{3/2}\tau_\varepsilon}{\gamma^2} > \sqrt{2\pi}$; as a result, in their setting, there are multiple equilibria when either τ_x or τ_ε is large. However, in my model with public disclosure, small supply noise alone could not generate multiplicity, as stated in the following proposition.

Proposition 3.3. Sufficiently small noise in supply (high τ_ε) alone does not give rise to multiple equilibria.

Proof. When noise in aggregate supply goes to zero, $\tau_\varepsilon \to \infty$, $\frac{\tau_y+\tau_p}{\sqrt{\tau_x}}$ becomes

$$\frac{1}{\sqrt{\tau_x}}\left(\tau_y + \frac{(\tau_x+\tau_y)^2}{\tau_y^2+\gamma^2/\tau_\varepsilon}\right) \to \frac{1}{\sqrt{\tau_x}}\left(\tau_y + \left(1+\frac{\tau_x}{\tau_y}\right)^2\right)$$

which is not guaranteed to be higher than $\sqrt{2\pi}$ without imposing assumptions on τ_y and τ_x.

The intuition behind the difference in the role of supply noise between this model and a model with only endogenous public information (i.e., price) is that, when there is exogenous public disclosure, public disclosure will also be incorporated into price, and the precision of price as an informative signal of the fundamental is no longer solely dependent on the precision of private information and the noise in supply. Instead, it is also conditioned on the precision of public disclosure. Holding the precision of public disclosure constant, if more noise is added to idiosyncratic private information, creditors will rationally put less weight on their private information and more weight on the public disclosure, thereby reducing its reliance of the price on private information and increasing the reliance on public disclosure. When private signals represent a lesser source of information, enhancing the private information alone will be inconsequential.

I then study the impact of public disclosure on the equilibrium properties of the coordination game. Prior studies have shown that when information is exogenous, multiplicity in equilibria can be prevented if private information is sufficiently precise relative to public information (Morris and Shin 1998); and when public information is endogenous, multiplicity in equilibria obtains if either private information is sufficiently precise or supply noise is sufficiently small (Angeletos and Werning 2006). However, in my model, there are two sources of public information: the exogenous public disclosure y, and the endogenous price p which aggregates idiosyncratic private signals. In this regard, my model is a mixture of Morris and Shin (1998) and Angeletos and Werning (2006), but the insights are distinct from both. The following proposition states the effect of public disclosure on the multiplicity of equilibria.

The following proposition states the upper boundary for the value of τ_y below which unique equilibrium could obtain.

Proposition 3.4. Unique equilibrium is possible if and only if $\tau_y \leq \tau_y^*$, where τ_y^* is the solution to the following pair of equations,

$$\begin{cases} \frac{2(\sqrt{2\pi\tau_x^*}-\tau_y^*)}{\tau_x^*+\tau_y^*} = \sqrt{\frac{\pi\tau_x^*}{2}} \\ \frac{1}{\sqrt{\tau_x^*}}\left(\tau_y^* + \frac{(\tau_x^*+\tau_y^*)^2}{\tau_y^{*2}+\gamma^2/\tau_\varepsilon}\right) = \sqrt{2\pi} \end{cases}$$

A graphical illustration of $\frac{1}{\sqrt{\tau_x}}\left(\tau_y + \frac{(\tau_x+\tau_y)^2}{\tau_y^2+\gamma^2/\tau_\varepsilon}\right) = \sqrt{2\pi}$ for the calibration $\gamma^2/\tau_\varepsilon = 0.2$ is given in Figure 3.3(a). Other parameter values for $\gamma^2/\tau_\varepsilon$ do not change the results in a qualitative way. The area below the $\frac{1}{\sqrt{\tau_x}}\left(\tau_y + \frac{(\tau_x+\tau_y)^2}{\tau_y^2+\gamma^2/\tau_\epsilon}\right) = \sqrt{2\pi}$ curve represents pairs of (τ_x, τ_y) which lead to a unique equilibrium, while the area above the curve represents pairs of (τ_x, τ_y) which lead to multiple equilibria. For sufficiently imprecise public disclosure (τ_y small), for example, $\tau_y' = 0.2$, there are two values of τ_x which make $\frac{1}{\sqrt{\tau_x}}\left(\tau_y' + \frac{(\tau_x+\tau_y')^2}{(\tau_y')^2+\gamma^2/\tau_\epsilon}\right) = \sqrt{2\pi}$ hold. Denote the two values by $\underline{\tau}_x(0.2)$ and $\overline{\tau}_x(0.2)$. There is a unique equilibrium for $\tau_x \in [\underline{\tau}_x(0.2), \overline{\tau}_x(0.2)]$, and multiple equilibria for $\tau_x \in (0, \underline{\tau}_x(0.2)) \cup (\overline{\tau}_x(0.2), \infty)$.

The main implication of Proposition 3.4 is that, multiplicity in the coordination game is neither prevented nor guaranteed by sufficiently precise private information, as in other studies such as Morris and Shin (2002) and Angeletos and Werning (2006). Instead, the relation between the existence of multiplicity and the precision of private information critically depends on the precision of public disclosure.

The following proposition characterizes the default outcome in the limiting cases.

Proposition 3.5. (i) For a given precision of public disclosure, $0 < \tau_y < \infty$, as private information becomes infinitely precise, $\tau_x \to \infty$, the default outcome is characterized by the following equilibria: (a) For $0 < p < 1$, $\theta^* \to \{0, p, 1\}$; (b) for $p \leq 0$, $\theta^* \to 1$; and (c) for $p \geq 1$, $\theta^* \to 0$.

(ii) For a given precision of private information, $0 < \tau_x < \infty$, as public disclosure becomes infinitely precise, $\tau_y \to \infty$, the default outcome is characterized by the following equilibria: (a) For $0 < y < 1$, $\theta^* \to \{0, y, 1\}$; (b) for $y \leq 0$, $\theta^* \to 1$; and (c) for $y \geq 1$, $\theta^* \to 0$.

The intuition behind Proposition 3.5 is that, when private information is infinitely precise, the coordinating role of price, which aggregates private information, is maximized. On the other hand, when public disclosure becomes infinitely precise, creditors will rely on public disclosure instead of price or private information to coordinate their actions.

In both cases, there is non-fundamental volatility in the default outcome, consistent with Angeletos and Werning (2006).

3.3 Expected Run Probability and Creditor Welfare

The previous section provides conditions for the determinacy of equilibrium, which is a fundamental result for models of multiple equilibria. A social planner, who is concerned with a utilitarian aggregator of creditors' payoff, may also want to prescribe a disclosure policy for public information, so that the probability of creditor run can be minimized, or creditors' ex ante welfare can be maximized. However, there is no intuitive answer to this question because of the multiplicity of equilibrium.

When equation (3.2.14) admits multiple (three) solutions for θ^*, the highest and lowest solutions represent stable equilibria, and the middle solution represents an unstable equilibrium. I numerically solve for the threshold levels, and calculate the expected welfare. Specifically, for each pair of (y, p), I obtain $\theta^*(y, p)$, and calculate the ex post probability of creditor run (an indicator of 0 or 1), as well as aggregate welfare as an equal-weighted sum of all creditors' utility.

I can then exhaust every possible realization of (y, p), and calculate the expected run probability as follows,

$$\begin{aligned} \text{EProb}(\tau_y; \tau_x, \tau_\varepsilon, \gamma, c) &= \int_y \int_p \mathbf{1}_{A(y,p)>\theta} dG(y,p) \\ &= \int_\eta \int_\varepsilon \mathbf{1}_{A(y,p)>\theta} d\Phi(\eta) d\Phi(\varepsilon) \end{aligned} \tag{3.3.1}$$

and calculate the expected welfare as follows,

$$\mathrm{EW}(\tau_y;\tau_x,\tau_\varepsilon,\gamma,c)=\int_\eta\int_\varepsilon[(\mathbf{1}_{A(y,p)\leq\theta}-c)(1-A(y,p))]d\Phi(\eta)d\Phi(\varepsilon) \tag{3.3.2}$$

where $A(y,p)=\Phi\big(\sqrt{\tau_x}(x^*(y,p)-\theta)\big)$ and $G(y,p)$ denotes the joint distribution of (y,p).

In calibrating the model, I am particularly interested in the case where increasing the precision of public disclosure moves the coordination game from unique equilibrium to multiple equilibria. Therefore, I calibrate the precision of private information to be a value in the range prescribed by Proposition 3.4 (e.g., $\tau_x = 0.3$). I also examine the case of high-quality private information endowment in which multiple equilibria obtain regardless of the precision of public disclosure (e.g., $\tau_x = 5$).

Figure 3.4 illustrates the comparative statics of expected run probability and creditor welfare with respect to the precision of public disclosure, where the solid line represents the equilibrium with highest threshold level of θ^* ("bad equilibrium") and the dashed line represents the equilibrium with lowest threshold level of θ^*("good equilibrium"). Panels (a) and (b) plot the ex ante run probability with respect to the precision of public disclosure, and Panels (c) and (d) plot the ex ante creditor welfare.

When private information is moderately precise so that public disclosure may be critical for equilibrium determinacy ($\tau_x = 0.3$, Panels (a) and (c)) there is unique equilibrium when public disclosure is sufficiently noisy, consistent with Proposition 3.2. As public disclosure gets more accurate, multiple equilibria begin to emerge and diverge from each other. In general, expected run probability monotonically increases with the precision of public disclosure if the bad equilibrium is at play, and the relationship exhibits an inverted-V shape if the "good equilibrium" is achieved. The impact of public disclosure on expected creditor welfare reflects the patterns in expected run probability: Creditor welfare is enhanced by public disclosure if good equilibrium is guaranteed, but may be reduced if bad equilibrium is played.

When private information is highly precise so that multiple equilibria always exist regardless of the precision of public disclosure ($\tau_x = 5$, Panels (b) and (d)), the role of public disclosure is even more nuanced. If the bad equilibrium is selected, expected run probability first decreases and then increases with the precision of public disclosure, and expected welfare first increases and then decreases with the precision of public disclosure. If instead the good equilibrium is selected, expected run probability first increases and then decreases with the precision of public disclosure, and expected welfare first decreases and then increases.

In sum, the welfare implications of public disclosure are far from unambiguous. More precise public disclosure could be undesirable from an ex ante perspective. The impact of public disclosure on expected probability of creditor run and creditor welfare critically depends on the private information of creditors,[4] and depends on which equilibrium is played if multiple equilibria exist.

3.4 Credit Derivatives

In the main model, the financial asset by creditors pays a dividend θ, and can be interpreted as firm's equity shares. In the following, I modify the payoff structure of the financial asset, so that it represents the payoff to credit derivatives. Take CDS for example. In CDS contracts, buyers are similar to buyers of insurance contracts, and they pay premiums over time. If the reference entity (the underlying company) does not default, buyers lose the premiums. If the company does default, the CDS allows the buyers to exchange the bonds of the company (which are worth significantly less than the case

[4]The observation that the role of public disclosure varies with the structure of private information is also made in theoretical studies on accounting disclosure in other settings, such as Bushman (1991).

of no default) for the principal amount of the bonds. Therefore, the payoff of a CDS is determined by the incidence of default, and the recovered value from default. Instead of assuming a binary payoff structure, I assume a continuous payoff which is related to the "financial distress" that the company faces.

Specifically, I assume that the payoff to the financial asset is $d = \theta - \Phi^{-1}(A)$, where A is the aggregate size of attack at date 2. As before, the dividend will be realized at the end of the second stage. The payoff $\theta - \Phi^{-1}(A)$ is negatively related to the probability of a credit event, which happens when $\theta < A$. The functional form of the payoff is chosen for tractability, and has no qualitative effect on the conclusions.

The information environment is the same as before. The firm makes a public disclosure, $y = \theta + \sigma_y \eta$, and creditors each receive an idiosyncratic noisy signal, $x_i = \theta + \sigma_x \xi_i$. To solve the model, suppose that creditors whose private information is below some threshold level $x^*(y,p)$ will decide not to rollover lending to the firm, or attack. As a result, the aggregate size of attack A is $A = \Phi\big(\sqrt{\tau_x}(x^*(y,p) - \theta)\big)$. Therefore, the payoff to the credit derivative can be written as

$$d(\theta, y, p) = (1 + \sqrt{\tau_x})\theta - \sqrt{\tau_x} x^*(y,p)$$

The price p of the asset is informationally equivalent to a linear transformation of price, $m(p) = \frac{p + \sqrt{\tau_x} x^*(y,p)}{1 + \sqrt{\tau_x}}$, which is the price of an asset that pays θ. Given that y is public information, observing $m(p)$ is equivalent to observing p. In the following, I work with $m(p)$ instead of p for algebraic simplicity. The expression and precision of $m(p)$ is given by the following lemma.

Lemma 3.2. The linear transformation of the price of the credit derivative, $m(p)$, is given by

$$m(p) = \theta + \frac{\tau_y \eta - \gamma(1+\sqrt{\tau_x})\sigma_\varepsilon \epsilon}{\tau_x + \tau_y} \tag{3.4.1}$$

and the precision of m as a signal of θ is given by

$$\tau_m \equiv \sigma_m^{-2} = \frac{(\tau_x + \tau_y)^2}{\tau_y^2 + \gamma^2(1+\sqrt{\tau_x})^2/\tau_\varepsilon} \tag{3.4.2}$$

The game at date 2 is similar as before, with the exception that the information set of creditors becomes (x_i, y, m). Assuming a linear monotone equilibrium with threshold rules: A creditor does not rollover lending whenever $x_i \leq x^*(y,m)$, and continues to lend when $x_i > x^*(y,m)$; corporate default happens when $\theta > \theta^*(y,m)$ and does not happen when $\theta \leq \theta^*(y,m)$. The thresholds are characterized by the following proposition.

Proposition 3.6. In a linear monotone equilibrium, the decisions threshold $x^*(y,m)$ and default threshold $\theta^*(y,m)$ are determined by the following two equations,

$$x^*(y,m) = \theta^* + \frac{1}{\sqrt{\tau_x}}\Phi^{-1}(\theta^*) \tag{3.4.3}$$

and

$$\frac{1}{\sqrt{\tau_x}}\Big(\tau_y y + \tau_m m - (\tau_y + \tau_m)\theta^*\Big) + \Phi^{-1}(\theta^*) - \sqrt{1 + \frac{\tau_y + \tau_m}{\tau_x}}\Phi^{-1}(c) = 0 \tag{3.4.4}$$

where $\tau_m = \frac{(\tau_x+\tau_y)^2}{\tau_y^2+\gamma^2(1+\sqrt{\tau_x})^2/\tau_\varepsilon}$.

Similar to the baseline case in which creditors trade equity shares, I derive conditions for the uniqueness/multiplicity of the equilibrium of the coordination game in which creditors trade credit derivatives instead.

Proposition 3.7. (Determinacy of equilibrium for the credit derivative case) (i) If

$\frac{\tau_y+\tau_m}{\sqrt{\tau_x}} \leq \sqrt{2\pi}$, or if $\frac{\tau_y+\tau_m}{\sqrt{\tau_x}} > \sqrt{2\pi}$ and $\tau_y y + \tau_m m \notin (\underline{t}, \bar{t})$, where

$$\bar{t} = (\tau_y + \tau_m)\bar{\theta} - \sqrt{\tau_x}\Phi^{-1}(\bar{\theta}) + \sqrt{\tau_x + \tau_y + \tau_m}\Phi^{-1}(c)$$

$$\underline{t} = (\tau_y + \tau_m)\underline{\theta} - \sqrt{\tau_x}\Phi^{-1}(\underline{\theta}) + \sqrt{\tau_x + \tau_y + \tau_m}\Phi^{-1}(c)$$

and $\bar{\theta} = \Phi\left(\left|\phi^{-1}\left(\frac{\sqrt{\tau_x}}{\tau_y+\tau_m}\right)\right|\right)$ and $\underline{\theta} = 1 - \bar{\theta}$, then there exists a unique equilibrium;

(ii) If $\frac{\tau_y+\tau_m}{\sqrt{\tau_x}} > \sqrt{2\pi}$ and $\tau_y y + \tau_m m \in (\underline{t}, \bar{t})$, then there exist multiple equilibria.

Proof. Analogous to Proposition 3.3.

As in the main model in which creditors trade equity shares, when creditors trade credit derivatives, similar insights obtain for the role of public disclosure in default outcomes.

Proposition 3.8. In the model where creditors trade credit derivatives, multiplicity obtains in a similar way to the model of equity trading. Specifically,

(i) Sufficiently small noise in supply (high τ_ε) alone does not lead to multiple equilibria.

(ii) There exists $\hat{\tau}_y$ such that for $\tau_y \in (0, \hat{\tau}_y)$, increasing the precision of private information τ_x may or may not lead to multiplicity; for $\tau_y \in [\hat{\tau}_y, \infty)$, multiple equilibria always obtain.

Proof. Analogous to Propositions 3.4 and 3.5.

Figure 3.3(b) presents a graphical illustration of $\frac{1}{\sqrt{\tau_x}}\left(\tau_y + \frac{(\tau_x+\tau_y)^2}{\tau_y^2+\gamma^2(1+\sqrt{\tau_x})^2/\tau_\varepsilon}\right) = \sqrt{2\pi}$. The interaction between public disclosure, private information, and multiplicity versus uniqueness of the equilibrium, is similar to the case of equity trading. In fact, the payoff structure is unlikely change the fundamental tradeoffs underlying the model: When public disclosure is sufficiently noisy (low precision), private information plays an more evident role in asset price, and therefore matters more for equilibrium outcomes; however, when public disclosure is sufficiently precise, creditors place less weight on private information in the process of price formation, and as a result, private information barely matters for determining multiplicity versus uniqueness of the coordination game.

3.5 Discussion: Endogenous and Exogenous Public Information

My findings provide new insights on the interaction between private information, public disclosure, and equilibrium outcomes in a global coordination game. The significance of these findings are better understood in the context of other studies on the role of information in related settings, in particular, Morris and Shin (1998; 2000), Angeletos and Werning (2006), and Hellwig, Mukherji, and Tsyvinski (2006).

In these studies and this paper, the ratio of the standard deviation of private signals to the variance of public signals determines the equilibrium of the coordination game. The distinguishing features of Morris and Shin, Angeletos and Werning, and this paper lie in the structure of public information. In Morris and Shin, there is a single exogenous public signal y. In Angeletos and Werning (2006), there is a single endogenous public signal—the asset price p which aggregates private information—and there is no other source of public information. In this paper, there are both exogenous and endogenous public signals: the public disclosure y and price p. As a consequence, my setting is more general than Morris and Shin and Angeletos and Werning, which can be obtained as special cases by dropping either source of public information. For example, in the main model, when there is no exogenous public disclosure, $\tau_y \rightarrow 0$,

$$\frac{1}{\sqrt{\tau_x}}\left(\tau_y + \frac{(\tau_x + \tau_y)^2}{\tau_y^2 + \gamma^2/\tau_\varepsilon}\right) \rightarrow \frac{\tau_x^{3/2}\tau_\varepsilon}{\gamma^2} \tag{3.5.1}$$

therefore the main result of Angeletos and Werning (2006) obtains as a special case of my setting.

The insights offered by my model, however, are distinct from the two special cases. In Morris and Shin, the equilibrium is unique if and only if private information is sufficiently precise relative to public information; In Angeletos and Werning, there are multiple equilibria if the private information is sufficiently precise or the noise in supply is small. However, in my setting, the role of private information critically depends on the quality of public disclosure: When public disclosure is precise, there are always multiple equilibria, and private information does not affect the equilibrium outcome; On the other hand, when public disclosure is sufficiently imprecise, enhancing the precision of private information may or may not lead to multiple equilibria. Specifically, there is a region of intermediate values for the precision of private information, within which unique equilibrium can be obtained. Outside this intermediate region, there are always multiple equilibria.

I do not claim to address all important questions regarding information aggregation and creditor runs, and I do not make sweeping policy proposals, such as prescriptive guidance on the optimal disclosure practice, based on the analysis. In models of multiple equilibria, model implications cannot be reduced to simple comparative statics. However, my results could potentially explain the large price volatility when public disclosure is highly precise (Proposition 3.5). In addition, precommitment to low level of precision of public disclosure can reduce the occurrences of multiple outcomes.

3.6 Concluding Remarks

This paper examines the role of public disclosure in a coordination game in which creditors first trade a financial asset of the firm, and then decide on whether to rollover lending to the firm. I show that public disclosure plays a critical role in determining equilibrium outcome of corporate default, through its interaction with private information and asset price. Public disclosure serves as an exogenous source of public information, while price constitutes an endogenous source of public information. I revisit the relationship between information precision and equilibrium multiplicity, and show that insights obtained in economies with exogenous or endogenous public information alone do not hold in my setting which has both exogenous *and* endogenous public information. I show that the insights apply to at least two broad categories of financial assets, equity shares and credit derivatives, which according to empirical evidence are widely traded by creditors. I also address the optimal disclosure policy to commit to from the perspective of a social planner who is concerned with creditors' ex ante welfare. This paper sheds lights on the role of public disclosure when there is already endogenous information in a coordination game.

I also study the welfare implications of public disclosure. I find that the impact of public disclosure on ex ante run probability and creditor welfare is conditioned on the precision of private information, and (in the case with multiplicity) which equilibrium is played. This result offers theoretical underpinnings for policy recommendations and disclosure mandates during times of crises. The main message is that ex ante, better disclosure is not unambiguously welfare-improving. Rather, whether it is socially desirable to require firms in financial distress to be more transparent depends on two factors: (i) whether creditors have good private knowledge; and (ii) which equilibrium could be played if multiple equilibria exist.

Even though I interpret the model as "creditor runs" for expositional convenience, there are other institutional settings in which analogous arguments can be made. In essence, creditor runs are similar to bank runs. While old-fashioned bank runs are caused by demand depositors when there is no deposit insurance, creditor runs are driven by short-term creditors who are concerned about the viability of the firm or other creditor beliefs of the viability of the firm. In this regard, this model can also be interpreted as depositors who trade a bank's equity shares. Taken further, in the context of political economy, it could also be interpreted as voters of a presidential candidate who also speculate on the likelihood of the candidate being elected in the prediction markets.

There are two important aspects of creditor trading that this paper does not address. First, creditor trading of financial assets of the borrowing firms is often based on the informational advantage that creditors have. Insider trading laws prohibit trading public securities on the basis of material non-public information. However, the large positions of credit derivatives held by financial institutions who lend to the reference entity of the derivative are not always regulated. The Dodd–Frank Wall Street Reform and Consumer Protection Act of 2010, though harsh on banks on many accounts, does not outlaw "risk-mitigating hedging activities" which include speculative trading by creditors. Future research could also examine the impact of corporate disclosure on insider trading by creditors.

Second, this paper is silent on the desirability of the trading of credit derivatives and other financial assets by creditors. Indeed, empirical studies have shown that the trading of credit derivatives may lower the cost of capital for borrowing firms (e.g., Saretto and Tookes 2010), and that use of credit derivatives is associated with greater supply of bank credit for large term loans (e.g., Hirtle 2008). Therefore, an interesting extension of the model is to incorporate cost of debt or credit supply, and study how they are influenced by public disclosure and credit derivative trading.

3.7 Appendix

Proof of Proposition 3.2. For notational ease, let $s \equiv \tau_y y + \tau_p p$. It is straightforward to show that $F(\cdot)$ is monotonic in θ^* for all s if and only if $\frac{\tau_y+\tau_p}{\sqrt{\tau_x}} \leq \sqrt{2\pi}$, and $F(\cdot)$ is non-monotonic in θ^* for all z if and only if $\frac{\tau_y+\tau_p}{\sqrt{\tau_x}} > \sqrt{2\pi}$.

I then characterize conditions for the realizations of exogenous public information (y) and endogenous public information (p) such that multiple solutions for θ^* obtain. In order for $F(\cdot)$ to have multiple solutions, I need to find realizations of s such that two local maximal/minimal points have the opposite signs, which requires that $s \in (\underline{s}, \bar{s})$, where

$$\bar{s} = (\tau_y + \tau_p)\bar{\theta} - \sqrt{\tau_x}\Phi^{-1}(\bar{\theta}) + \sqrt{\tau_x + \tau_y + \tau_p}\Phi^{-1}(c)$$

$$\underline{s} = (\tau_y + \tau_p)\underline{\theta} - \sqrt{\tau_x}\Phi^{-1}(\underline{\theta}) + \sqrt{\tau_x + \tau_y + \tau_p}\Phi^{-1}(c)$$

and $\bar{\theta} = \Phi\left(\left|\phi^{-1}\left(\frac{\sqrt{\tau_x}}{\tau_y+\tau_p}\right)\right|\right)$ and $\underline{\theta} = 1 - \bar{\theta}$. Note that $(\bar{\theta}, \underline{\theta})$ denotes the two values of θ where the slope of $F(\cdot)$ is zero.

Proof of Proposition 3.4. Define

$$F(\tau_x, \tau_y) \equiv \frac{1}{\sqrt{\tau_x}}\Big(\tau_y + \frac{(\tau_x + \tau_y)^2}{\tau_y^2 + \gamma^2/\tau_\varepsilon}\Big) - \sqrt{2\pi} \tag{3.7.1}$$

and let $F_x(\tau_x, \tau_y) \equiv \frac{\partial F}{\partial \tau_x}$ and $F_y(\tau_x, \tau_y) \equiv \frac{\partial F}{\partial \tau_y}$. Consider (τ_x^0, τ_y^0) such that $F(\tau_x^0, \tau_y^0) = 0$, which is the equation that characterizes the boundary of multiplicity versus uniqueness. Suppose $F(\tau_x^0, \tau_y^0) = 0$ is equivalent to $\tau_y^0 = \tau_y^0(\tau_x^0)$. By the implicit function theorem, the slope of $\tau_y^0 = \tau_y^0(\tau_x^0)$ is given by

$$\frac{d\tau_y^0(\tau_x^0)}{d\tau_x^0} = -\frac{F_x(\tau_x^0,\tau_y^0)}{F_y(\tau_x^0,\tau_y^0)}$$
$$= \frac{\sqrt{\tau_x}\Big(-\frac{2(\tau_x+\tau_y)}{\sqrt{\tau_x}(\gamma^2/\tau_\epsilon+\tau_y^2)}+\frac{1}{2\tau_x^{3/2}}\big(\tau_y+\frac{(\tau_x+\tau_y)^2}{\gamma^2/\tau_\epsilon+\tau_y^2}\big)\Big)}{1-\frac{2\tau_y(\tau_x+\tau_y)^2}{(\gamma^2/\tau_\epsilon+\tau_y^2)^2}+\frac{2(\tau_x+\tau_y)}{\gamma^2/\tau_\epsilon+\tau_y^2}}\Bigg|_{(\tau_x^0,\tau_y^0)} \tag{3.7.2}$$

Note that $F(\tau_x^0,\tau_y^0)=0$ implies that $\frac{(\tau_x^0+\tau_y^0)^2}{(\tau_y^0)^2+\gamma^2/\tau_\epsilon} = \sqrt{2\pi\tau_x^0}-\tau_y^0$. Therefore, (3.7.2) can be rewritten as

$$\frac{d\tau_y^0(\tau_x^0)}{d\tau_x^0} = \frac{-\frac{2(\sqrt{2\pi\tau_x^0}-\tau_y^0)}{\tau_x^0+\tau_y^0}+\sqrt{\frac{\pi\tau_x^0}{2}}}{1-2\tau_y^0\big(\frac{\sqrt{2\pi\tau_x^0}-\tau_y^0}{\tau_x^0+\tau_y^0}\big)^2+2\frac{\sqrt{2\pi\tau_x^0}-\tau_y^0}{\tau_x^0+\tau_y^0}} \tag{3.7.3}$$

$\frac{d\tau_y^0(\tau_x^0)}{d\tau_x^0}=0$ characterizes the maximum value for τ_y^0, which leads to the system of equations in Proposition 3.4. It can be shown that this is a decreasing function in τ_x^0, i.e., $\frac{d^2\tau_y^0(\tau_x^0)}{d(\tau_x^0)^2}<0$ and there exists $\hat{\tau}_x$ such that $\frac{d\tau_y^0}{d\tau_x^0}|_{\hat{\tau}_x}=0$.

Proof of Proposition 3.5. Equation (3.2.14) can be written as

$$-\theta^* + \frac{\sqrt{\tau_x}}{\tau_y+\tau_p}\Phi^{-1}(\theta^*) = \frac{\sqrt{\tau_x+\tau_y+\tau_p}}{\tau_y+\tau_p}\Phi^{-1}(c) - \frac{\tau_y}{\tau_y+\tau_p}y - \frac{\tau_p}{\tau_y+\tau_p}p \tag{3.7.4}$$

(i) For a given level of τ_y, as $\tau_x\to\infty$,

$$\tau_p = \frac{(\tau_x+\tau_y)^2}{\tau_y^2+\gamma^2/\tau_\epsilon}\to\infty$$

By L'Hopital's rule,

$$\frac{\sqrt{\tau_x}}{\tau_y+\tau_p}\to 0$$

$$\frac{\sqrt{\tau_x+\tau_y+\tau_p}}{\tau_y+\tau_p}\to 0$$

$$\frac{\tau_y}{\tau_y + \tau_p} \to 0$$

$$\frac{\tau_p}{\tau_y + \tau_p} \to 1$$

Therefore, for $0 < \theta^* < 1$, in the limit of $\tau_x \to \infty$, equation (3.2.14) is equivalent to

$$\theta^* = p \tag{3.7.5}$$

In addition, we have $\lim_{\theta^* \to 0} \Phi^{-1}(\theta^*) \to -\infty$ and $\lim_{\theta^* \to 1} \Phi^{-1}(\theta^*) \to \infty$.

Therefore, as $\tau_x \to \infty$, the coordination game is characterized by the following equilibria: (a) For $0 < p < 1$, $\theta^* \to \{0, p, 1\}$; (b) for $p \leq 0$, $\theta^* \to 1$; and (c) for $p \geq 1$, $\theta^* \to 0$.

(ii) For a given level of τ_x, as $\tau_y \to \infty$,

$$\tau_p = \frac{(\tau_x + \tau_y)^2}{\tau_y^2 + \gamma^2/\tau_\epsilon} \to 1$$

By L'Hopital's rule,

$$\frac{\sqrt{\tau_x}}{\tau_y + \tau_p} \to 0$$

$$\frac{\sqrt{\tau_x + \tau_y + \tau_p}}{\tau_y + \tau_p} \to 0$$

$$\frac{\tau_y}{\tau_y + \tau_p} \to 1$$

$$\frac{\tau_p}{\tau_y + \tau_p} \to 0$$

Therefore, for $0 < \theta^* < 1$, in the limit of $\tau_x \to \infty$, equation (3.2.14) is equivalent to

$$\theta^* = y \tag{3.7.6}$$

In addition, we know that $\lim_{\theta^* \to 0} \Phi^{-1}(\theta^*) \to -\infty$ and $\lim_{\theta^* \to 1} \Phi^{-1}(\theta^*) \to \infty$.

Therefore, as $\tau_x \to \infty$, the coordination game is characterized by the following equilibria: (a) For $0 < y < 1$, $\theta^* \to \{0, y, 1\}$; (b) for $y \leq 0$, $\theta^* \to 1$; and (c) for $y \geq 1$, $\theta^* \to 0$.

Proof of Lemma 3.2. Suppose $m(p)$ is a noisy signal of θ with variance σ_m^2 and precision $\tau_m = \sigma_m^{-2}$, where the variance and precision will be determined. Conditional on observing x_i, y, and $m(p)$, creditor i's expectation of θ is given by $E[d|(x_i, y, m)] = (1+\sqrt{\tau_x})\frac{\tau_x x_i + \tau_y y + \tau_m m}{\tau_x + \tau_y + \tau_m} - \sqrt{\tau_x} x^*(y,p)$ and $\text{var}[d|(x_i, y, m)] = \frac{(1+\sqrt{\tau_x})^2}{\tau_x + \tau_y + \tau_m}$. Therefore, the demand of creditor i is given by

$$
\begin{aligned}
k_i(x_i, y, m) &= \frac{E[d|(x_i, y, m)] - p}{\gamma \text{var}[d|(x_i, y, m)]} \\
&= \frac{(1+\sqrt{\tau_x})\frac{\tau_x x_i + \tau_y y + \tau_m m}{\tau_x + \tau_y + \tau_m} - \sqrt{\tau_x} x^*(y,p) - \left((1+\sqrt{\tau_x})m - \sqrt{\tau_x} x^*(y,p)\right)}{\gamma \frac{(1+\sqrt{\tau_x})^2}{\tau_x + \tau_y + \tau_m}} \\
&= \frac{\tau_x(x_i - m) + \tau_y(y - m)}{\gamma(1+\sqrt{\tau_x})} \qquad (3.7.7)
\end{aligned}
$$

and the aggregate demand is

$$K(\theta, y, m) = \frac{\tau_x(\theta - m) + \tau_y(y - m)}{\gamma(1+\sqrt{\tau_x})} \qquad (3.7.8)$$

By market clearing condition (3.2.3), we have $\frac{\tau_x(\theta - m) + \tau_y(y - m)}{\gamma(1+\sqrt{\tau_x})} = \sigma_\varepsilon \varepsilon$, and it follows that

$$m(p) = \frac{p + \sqrt{\tau_x} x^*(y,p)}{1+\sqrt{\tau_x}} = \theta + \frac{\tau_y \eta - \gamma(1+\sqrt{\tau_x})\sigma_\varepsilon \varepsilon}{\tau_x + \tau_y} \qquad (3.7.9)$$

It is then straightforward to obtain the precision of price.

3.8 References

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Figure 3.1: Bank Lending and CDS Position

This figure presents the aggregate loans and CDS positions of large US banks (banks with at least $1 billion in assets in 2003). The data is based on Table 1 of Duffie (2008), which is compiled from the Federal Reserve Bank of Chicago's bank holding company data, 2001-2006.

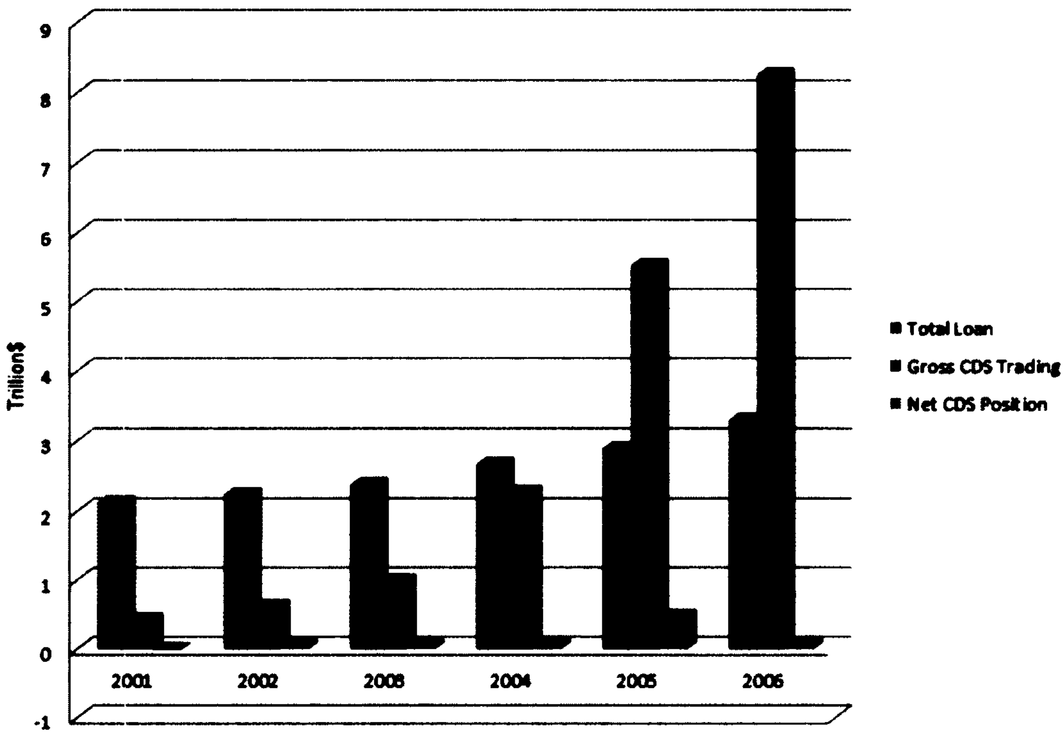


Figure 3.2: Timeline of Bankruptcy

This figure reproduces Figure 1 of Ivashina et al. (2012). As the figure illustrates, the bankruptcy process typically involves the following steps: (i)"a petition in the federal bankruptcy court in the bankruptcy district (in which the firm is either headquartered, incorporated, or in which the firm does a significant amount of business)"; (ii) "shortly after filing the petition, the debtor is required to file its schedules of assets and liabilities"; (iii) trading and claims transfer by creditors; (iv) "voting for the [restructuring] plan takes place through a balloting process managed by the restructuring and insolvency administrators"; (v) exit.

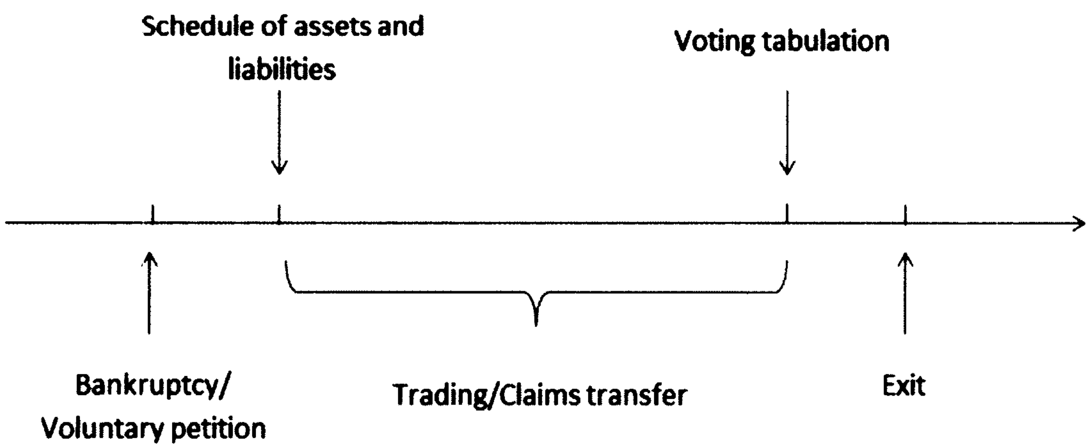
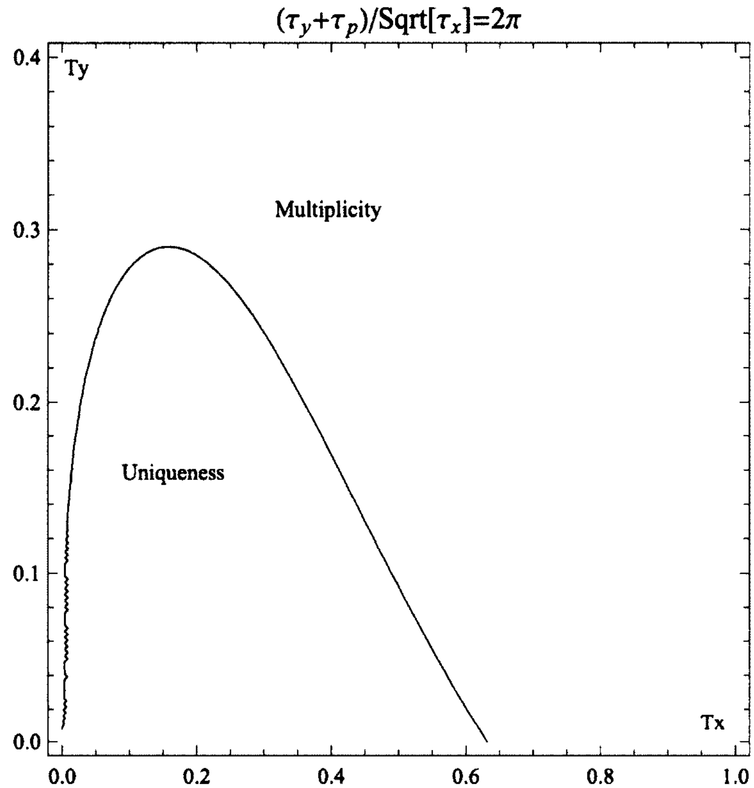
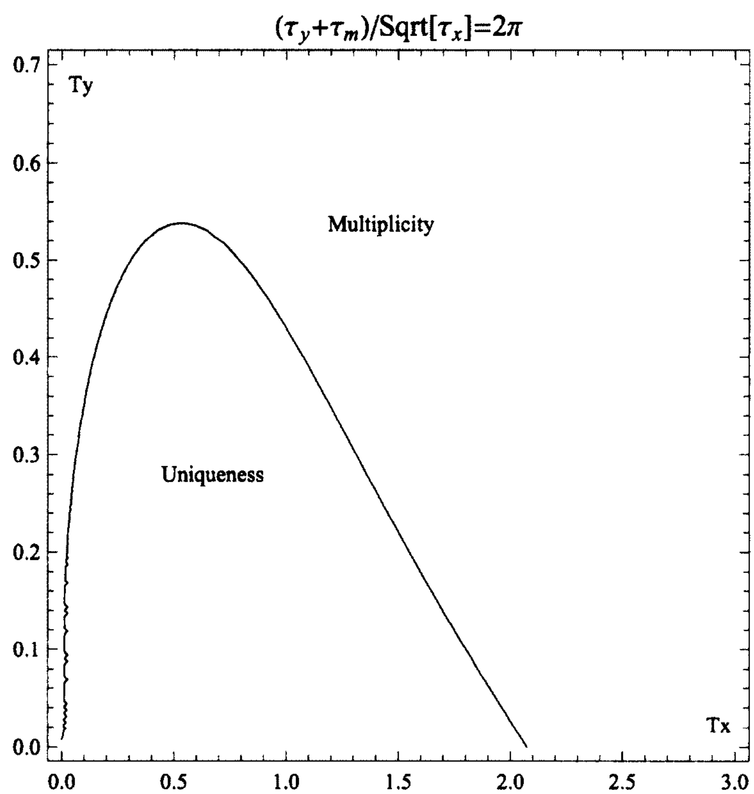


Figure 3.3: Determinacy of Equilibrium

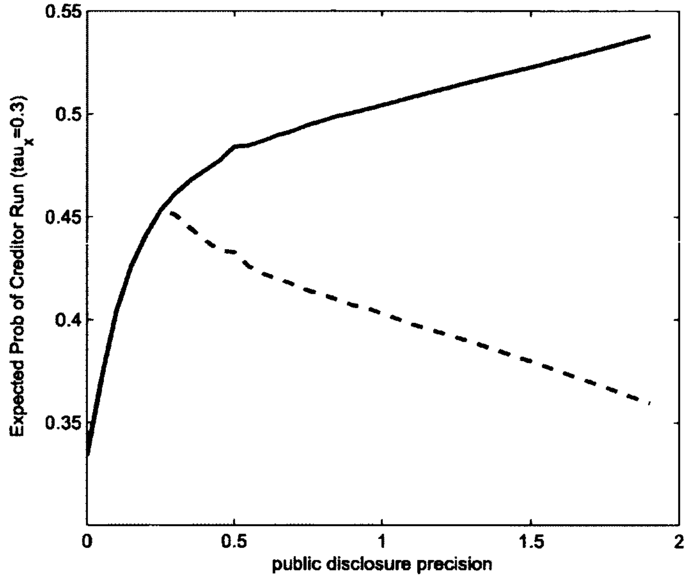


(a) Model with equity traded

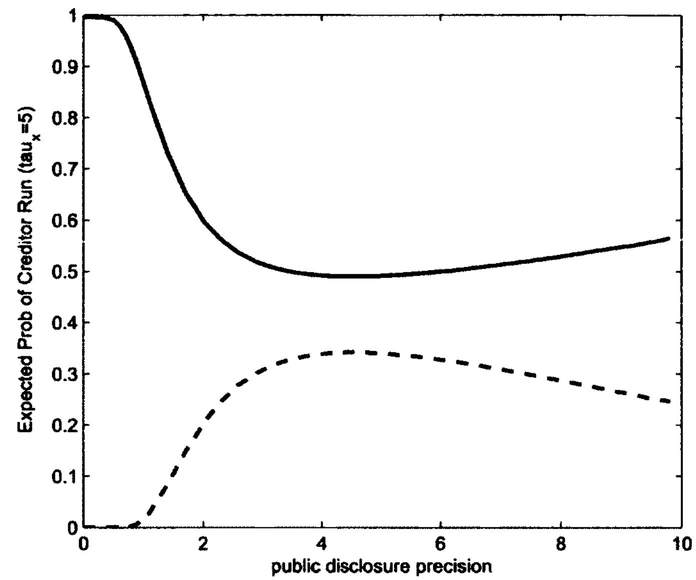


(b) Model with credit derivative traded

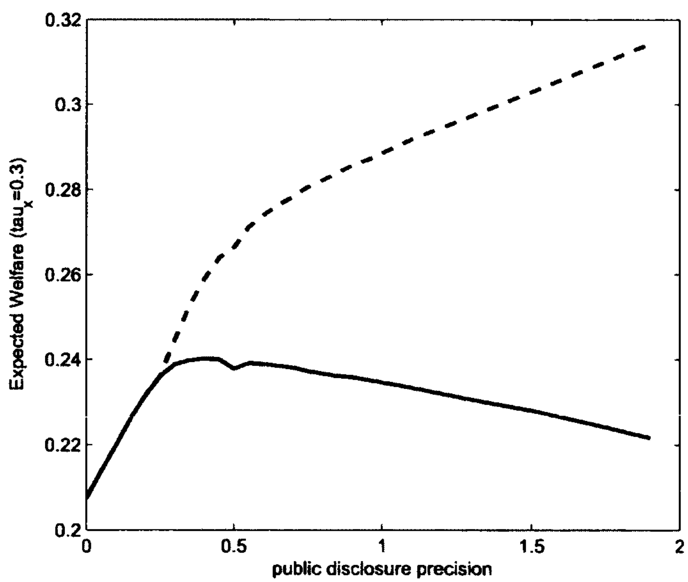
Figure 3.4: Expected Run Probability and Creditor Welfare



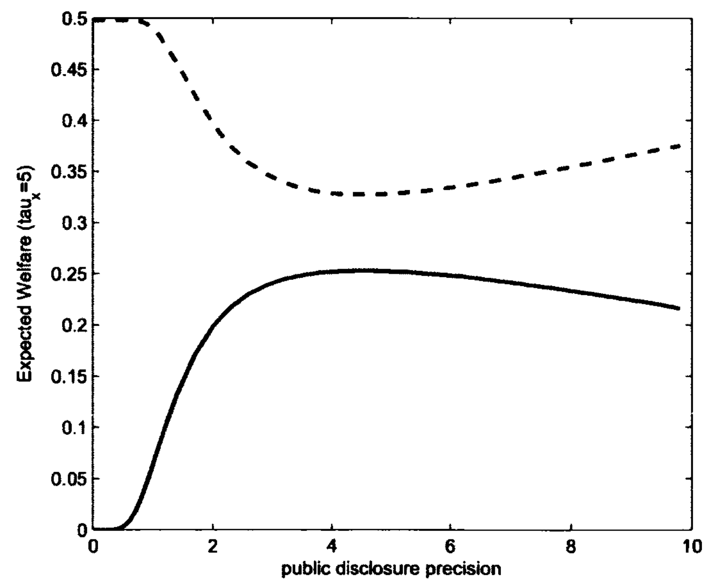
(a) Expected run probability ($\tau_x = 0.3$)



(b) Expected run probability ($\tau_x = 5$)



(c) Expected creditor welfare ($\tau_x = 0.3$)



(d) Expected creditor welfare ($\tau_x = 5$)

Chapter 4

Demand Deposits, Loan Loss Provision, and Risk Sharing

4.1 Introduction

Banks raise funds from depositors who face liquidity shocks, promise them with demandable deposit contracts, and invest in risky long-term assets, or loans. By transforming illiquid assets into liquid liabilities, banks essentially provide insurance arrangements in which depositors share the risk of liquidating the long-term asset early at a loss. Such risk sharing role of banks is the key reason for their existence in the first place. However, bank regulators have overemphasized the importance of insolvency avoidance while neglecting the role of risk sharing. In this paper, I show that loan loss provision, which is embraced by regulators as a tool to reduce pro-cyclicality and insolvency, may lead to suboptimal risk sharing by introducing undesirable contingency to demand-deposit contracts.

Bank loan loss provision plays two roles: informational and regulatory. On one hand, loan loss provision is indicative of the quality of loans; on the other hand, loan loss provision is a form of regulatory capital. Because of its significance, accounting for bank loan loss provision has been at the center of a recent regulatory debate between bank regulators and accountants. While bank regulators blame incurred loss model for amplifying the financial downturn spiral and causing pro-cyclicality, accountants and accounting researchers downplay the call for counter-cyclical or more forward-looking loan loss provisioning, arguing that financial reporting has a different objective from bank regulation, and that granting more discretion to loan loss accounting would give rise to more opportunism. Is there theoretical basis for regulators' push for more counter-cyclical loan loss provision, and for accountants' adherence to the informational role of provisions?

Despite of the debate between regulators and accountants, there is a surprising dearth of formal treatise of the role of loan loss provision in banks. Banking theories are largely developed in an institutional vacuum deprived of accounting for loan losses; on the other hand, empirical accounting studies focus on loan loss provision as a tool for earnings management or capital management. None has examined the effect of loan loss provision on the risk-sharing role of banks, even though such role is pivotal to the mere existence of banks.

Motivated by the imbalance between abundant policy discourse and scarce formal theorization, this essay develops a framework for studying the role of loan loss provision in a canonical banking model. Building on a variant of the Diamond and Dybvig (1983) model with fundamental uncertainty, I assume that at the intermediate date, a noisy signal about loan payoff becomes public. Even though such signal may be uninformative, bank management is required to make a loan loss provision consistent with the change in expected payoff as implied by this signal. I solve for the optimal demand-deposit contracts contingent on loan loss provision, and show that such contracts induce underinvestment in the risky loan, and lead to lower expected utility for depositors, relative to two benchmark cases where there is either no interim information or perfectly accurate interim information.

Therefore, in the presence of liquidity risk, loan loss provision may introduce undesirable contingency on demand-deposit to the detriment of risk sharing.

This paper contributes to banking theories and accounting literature in several respects. First, I propose a tractable framework for studying the role of loan loss provision in banks which transform illiquid assets into liquid liabilities. Second, this study adds to the growing line of research on the interplay between credit risk and liquidity risk. Third, at a time when regulatory endeavors fixate on how to reduce procyclicality (e.g., Brunnermeier et al. 2009), my study underscores the risk sharing function of banks. In other words, this paper offers theoretical underpinning for accountants' adherence to prioritizing the informational role of loan loss provision, in spite of regulators' campaign for counter-cyclical loan loss provision.

The remainder of the paper is structured as follows. Section 4.2 outlines the main arguments in the regulatory debate on accounting methods for loan loss provision, and reviews bank theories and empirical research on loan loss provision as a literature context for the current study. Section 4.3 presents the basic model setup and solves it for the benchmark case where there is no interim information or loan loss provision. Section 4.4 introduces loan loss provision to the model, and studies two forms of loan loss provision: perfectly informative and noisy. I solve for the incentive-compatible demand-deposit contract and the optimal investment decision, conduct a calibrated comparison of welfare across different information structures, and discuss the implications for the current regulatory debate. Section 4.5 concludes.

4.2 Background and Literature

4.2.1 The regulatory debate

Accounting for bank loan loss provision has been blamed for amplifying the financial downturn spiral, and is at the center of a recent regulatory debate between bank regulators and accountants.

Bank regulators argue that although "this aspect of accounting [loan loss provisioning] has major implications for the soundness of banks,"[1] loan loss reserve has received insufficient attention compared with Tier-1 capital reserve. For example, Dugan (2009) reiterates the importance of being aware of the fact that "loan loss reserve is part of capital." More recently, along with the Basel III deliberation process, bank regulators have been campaigning for revised accounting rules of loan loss provision to address pro-cyclicality concerns. The Basel Committee on Banking Supervision was asked by its oversight board to ensure the creation of a robust provisioning method based on expected losses. Indeed, in the Basel III reform package that ensues, mitigating pro-cyclicality is one of the principal objectives intended to achieve, and the Basel Committee has envisioned concerted measures which include more forward-looking loan loss provisioning and a counter-cyclical capital buffer (Risk 2010b).

Accountants and accounting researchers, on the other hand, contend that financial reporting has a different objective from bank regulation (Benston and Wall 2005; Barth and Landsman 2010; Bushman and Landsman 2010), and that "accounts aren't written specifically for regulators-they are general-purpose financial statements for the benefit

[1]Remark by an anonymous senior regulator, cited by Risk (2010a). Indeed, according to the current definition of "regulatory capital", general loan loss reserve is part of Tier-2 capital. The sum of Tier-1 and Tier-2 capital represents the numerator of the solvency ratio and needs to meet minimum regulatory requirements.

of users including creditors, regulators, tax authorities, investors, analysts, the public, customers and employees."[2] There are also concerns that granting more discretion to loan loss accounting would give rise to more opportunism in managerial reporting behavior (Ryan 2007, Chapter 5). In particular, bank managers may utilize less hard-line loan loss models to smooth earnings, therefore reducing the information content of loan losses. In general, the accounting community contends that the informational role of loan loss provision should dominate all other purported uses such as capital regulation.

4.2.2 Prior literature

The regulatory debate has found limited theoretical underpinning in the finance and accounting literature. In order to inform policy deliberations on capital regulation, a banking theory needs to incorporate the essence of accounting for loan loss provision. Unfortunately, the two streams of literature, i.e., bank theories and empirical studies on bank loan loss provision, have evolved in isolation from each other. In the following, I selectively review each line of research.

Banking theory

The micro-orientated literature on liquidity risk and reserve management is often from a single bank's perspective and does not touch on a broader risk sharing problem (see Freixas and Rochet 2007 for a synthesis). Even though the concept of "reserve" for liquidity risk could readily be extended to loan loss reserve (reserve for credit risk), my focus is the effect of "reserve for credit risk" on efficient risk sharing among depositors who experience liquidity risk.

My study builds on a version of the Diamond and Dybvig (1983) model, and is most

[2]Remarks by Colin Martin, partner in the financial services technical advisory team at KPMG in London, cited in Risk (2010a).

closely related to Jacklin and Bhattacharya (1988) and Allen and Gale (1998).[3] In a model with fundamental uncertainty and smooth preference, Jacklin and Bhattacharya (1988) show that an interim signal about the risky payoff could lead to informational run, a phenomenon distinct from the "sunspot" view of bank runs à la Diamond and Dybvig (1983). Allen and Gale (1998) show that since real world deposit contracts cannot be ex ante contingent on the risky payoff, bank runs may provide desirable contingency and help achieve first-best risk sharing. These models are in line with the business cycle view of financial crisis (see empirical evidence of Gorton (1988)), rather than the sunspot view (Diamond and Dybvig 1983), in which there is no aggregate uncertainty and banks runs occur solely because late consumers' beliefs.

Even though I do not explicitly model bank insolvency, the mere existence of loan loss provision has been intended to reduce insolvency risk. In this regard, this paper is also related to several papers that collectively study liquidity risk and insolvency risk. In particular, Diamond and Rajan (2005) show that liquidity and solvency problems are interrelated. Morris and Shin (2009) decompose total credit risk into illiquidity risk and asset insolvency risk.

This study is also related to theories of bank capital regulation, such as Dewatripont and Tirole (1994), Gale (2003; 2004), Gale and Özgür (2005), Allen and Gale (2003; 2007, Chapter 7). In particular, Allen and Gale propose a model with a group of risk neutral investors who own capital, and show that optimal capital structure improves risk sharing between shareholders and depositors. In contrast, my paper does not explicitly model bank capital, but studies the conventional risk sharing problem between depositors. In addition, Gorton and Winton (2000) show that a system-wide increase in bank capital induces forces

[3] This framework has also been adopted to allow incomplete information, e.g., Goldstein and Pauzner (2005).

consumers to make less bank deposits and hold more bank equity, jeopardizing liquidity creation. Fahri, Golosov and Tsyvinski (2009) provide a mechanism design theory of liquidity regulation for banks.

Empirical studies on loan loss provision

One strand of empirical accounting literature focuses on the informational content (informational role) of bank loan loss provision and related disclosure. For example, Wahlen (1994) notes that bank financial statements provide three separate disclosure of default risks: non-performing loans, loan loss provisions, and loan chargeoffs. Beaver et al. (1989) and Elliott et al. (1991) document a positive relation between stock returns and loan loss provision. Liu and Ryan (1995) examine the implication of loan portfolio composition on the nature of information contained in loan loss provision.

Another strand of accounting research examines managerial incentives to adjust accounting measures used by regulators, focusing on earnings management and capital management hypotheses, such as McNichols and Wilson (1988), Moyer (1990), Beaver and Engel (1996), Ahmed et al. (1999), and Perez et al. (2008).

Other studies have examined whether loan loss provision is pro-cyclical or contributes to the pro-cyclicality of bank activities. Laeven and Majnoni (2003) find that many banks' loan loss provisions are untimely relative to the business cycle, which magnifies the impact of the economic cycle on banks' income and capital. Recent studies have compared the implications of different loan loss provision schemes that are used in practice or proposed as the new norm, such as the incurred loss model, the expected loss model, the fair value model, and the Spanish dynamic provisioning model (De Lis et al. 2001). In particular, Beatty and Liao (2011) provide empirical support that banks with forward-looking provisioning display less pro-cyclicality of lending activities in recessions; Bushman and

Williams (2009) find that banks in high discretion regimes (where the discretion measure is based on loan loss provision practices) exhibit less transparency and more risk-shifting relative to banks with less discretion.

4.3 The Basic Model

4.3.1 Model setup

I study a version of the canonical model of financial intermediation developed by Diamond and Dybvig (1983) and also found in Jacklin (1987), Jacklin and Bhattacharya (1988), Allen and Gale (1998; 2004), and Farhi, Golosov, and Tsyvinski (2009), among many other studies. There are two key features in my model. First, similar to Jacklin and Bhattacharya (1988) and Allen and Gale (1998), the payoff to the long-term investment technology is uncertain. The second feature is that I explicitly model loan loss provision which in turn has to be consistent with an interim signal.

Financial intermediary and technology. There are three dates $t = 0, 1, 2$ and a single, all-purpose commodity at each date that can either be invested or consumed. Investment decisions are made by a financial intermediary which I call a bank. The bank can make two kinds of investment. A short-term storage technology converts one unit of good at date 0 to one unit at date 1, and a long-term investment (which I call a risky "loan") that pays $\tilde{R}$ at date 2 for each unit invested at date 0. The return to the long-term investment is uncertain and will depend on the state of the world, $s \in \{H, L\}$,

$$\tilde{R} = \begin{cases} R & \text{w. prob. } p \\ 0 & \text{w. prob. } 1 - p \end{cases}$$

where $R > 1$. If liquidated at the intermediate date (date 1), the investment generates

1. Therefore, the long-term investment is entirely irreversible. Banks have access to both investment technologies, but agents as will be introduced later only have access to the storage technology. Banks collect deposits from agents and make investment decisions. I assume that there are a large number of banks, therefore perfect competition will drive bank profit to zero, and each bank will offer deposit contracts that maximize the expected utility of depositors. Therefore, the optimization problem that a representative bank solves is an optimal risk-sharing problem. In other words, the bank behaves as a benevolent social planner. In the following I study the portfolio choice and the design of optimal demand-deposit contracts by a representative bank.

Depositors and preference shocks. There are a continuum of agents (depositors). Each agent has an initial endowment of one unit of the consumption good at date 0 and none at subsequent dates. Ex ante (at date 0), agents are identical and uncertain about their types, which will be privately observed at date 1 as an idiosyncratic preference shock.

$$U(c_1, c_2) = \begin{cases} u(c_1) & \text{w. prob. } \pi \\ u(c_2) & \text{w. prob. } 1 - \pi \end{cases}$$

where c_t denotes consumption at date t. Agents who only value consumption at date 1 (date 2) are called early consumers (late consumers). An alternative formulation of agents' utility is

$$U(c_1, c_2; \theta) = (1 - \theta)u(c_1) + \theta u(c_2) \tag{4.3.1}$$

where $\theta \in \{0, 1\}$ is agent's type (idiosyncratic preference shock). Preference shocks are i.i.d. across agents. Because the number of agents is large, the Law of Large Numbers dictates that the fraction of type-1 agents is the same as the probability of becoming a type-1 agent, which is π. Similarly, the fraction of type-2 agents is $1 - \pi$. In other words, there is no aggregate uncertainty about the fraction of early (late) consumers.

The main analysis is intended to be generally applicable to various utility functions. However, for some parts of the paper, I assume one of the following two utility functions, (i) logarithmic utility function,

$$u(c) = \ln(c)$$

(ii) exponential utility function,

$$u(c) = -\frac{1}{\gamma}\exp(-\gamma c)$$

The choice of log utility function is for ease of illustration of some benchmark cases; The choice of exponential utility function allows for a closed-form solution for demand-deposit contracts as a function of loan loss provision. As will become clear later, this is because exponential utility function renders the incentive compatibility constraints in bank's optimization problem dispensable.

"Depositors" in the model should not be narrowly interpreted as depositors in a conventional banking system. Rather, "depositor" may refer to any supplier of short-term capital. For example, in the context of securitized banking where repo agreements serve as the main source of funds, demand deposits in my model may well be interpreted as repo transactions, and depositors can be interpreted as lenders of repo. Consequently, runs by depositors could take the form of runs on repo: When information reveals that the risky asset used as collateral is going to be of really low value, lenders may impose a larger haircut on the collateral. This is analogous to informational runs à la Jacklin and Bhattacharya (1988). For another example, money market funds perform maturity transformation by investing in long-term assets while offering investors the ability to withdraw funds on demand. Such examples are abundant as financial intermediation functions traditionally associated with banking have been increasingly performed by nonbank institutions.

Uncertainty and demand deposit. At date 1, uncertainty about depositors' liquidity preferences is resolved and privately revealed to depositors themselves, but uncertainty about returns to the risky project may not be fully resolved. I will consider alternative model specifications with different information structure.

A demand deposit is a contract that requires a depositor to deposit one unit of his endowment at date 0 in exchange for the right to withdraw a pre-specified amount, conditional on the self-reported type. The form of feasible demand-deposit contract critically depends on the information set of agents. For example, in the benchmark model, when there is no information at date 1 about risky loan payoffs, the consumption of type-2 agents will be contingent on the realized loan payoff, (c_1, c_{2H}, c_{2L}), where c_1 is the consumption (withdrawal) of agents who claim to be type 1 at date 1, c_{2H} (c_{2L}) is the date-2 consumption of agents who claim to be type 2, conditional on the risky loan payoff turns out to be R_H (R_L).

4.3.2 No interim information

Observable types

I then consider the benchmark case without interim information by solving a bank's problem in which agents' types are common knowledge after they are realized at date 1, and the bank is only constrained by the resource (feasibility) constraint. Because there is no interim signal about loan payoff at date 1, the payoff to the risky loan is not known until date 2, and the bank does not need to make any loan loss provision. The demand-deposit contract offered by the bank is assumed to take the form $c \equiv (c_1, c_{2H}, c_{2L})$. As Jacklin (1987) argues, this kind of contract optimally combines the two types of deposits that real world depositors may hold: a long-term (2-period) savings deposit and a short-term

(1-period) checking account which is similar to the demand deposit contract in Diamond and Dybvig (1983).

The bank chooses deposit contract and makes investment in storage and loan technology to maximize the expected utility of agents. Because there is a continuum 1 of agents, the objective function can either be interpreted as the expected utility of a representative agent, or the summation of utility of all agents. The bank chooses deposit contracts c and makes investment decision L to solve the following optimization problem, which I call SP^1,

$$\max_{c} \pi u(c_1) + (1-\pi)\Big((1-p)u(c_{2H}) + pu(c_{2L})\Big) \tag{4.3.2}$$

subject to

$$\pi c_1 + (1-\pi)\frac{c_{2H}}{R_H} = 1 \tag{4.3.3}$$

$$\pi c_1 + (1-\pi)\frac{c_{2L}}{R_L} = 1 \tag{4.3.4}$$

Plug in $c_{2H} = \frac{R_H(1-\pi c_1)}{1-\pi}$ and $c_{2L} = \frac{R_L(1-\pi c_1)}{1-\pi}$, the first order condition with respect to c_1 is

$$u'(c_1) = (1-p)R_H u'(c_{2H}) + pR_L u'(c_{2L}) \tag{4.3.5}$$

Therefore, the optimal c_1 is the solution to the following equation,

$$u'(c_1) = (1-p)R_H u'\Big(\frac{R_H(1-\pi c_1)}{1-\pi}\Big) + pR_L u'\Big(\frac{R_L(1-\pi c_1)}{1-\pi}\Big) \tag{4.3.6}$$

If I work with log utility function, $u(c) = \ln(c)$, I can even get closed form solution,

$$\frac{1}{c_1} = (1-p)R_H \frac{1-\pi}{R_H(1-\pi c_1)} + pR_L \frac{1-\pi}{R_L(1-\pi c_1)} \tag{4.3.7}$$

$$\frac{1}{c_1} = \frac{1-\pi}{1-\pi c_1} \tag{4.3.8}$$

Therefore, $c_1 = 1$, $c_{2H} = R_H$, and $c_{2L} = R_L$. Investment in short asset is $L = \pi$. The maximized value of expected utility is

$$EU^{\text{no, Log}} = (1-\pi)\Big((1-p)\ln(R_H) + p\ln(R_L)\Big)$$

It is easy to argue that perfect risk sharing achieves higher expected utility than the case of autarky. The intuition is that, agents in autarky will inevitably regret for overinvesting or underinvesting in long-term loans once they learn their types at date 1; while in the presence of banks, agents can pool their individual liquidity risk and make more efficient aggregate investment decisions.

Unobservable types

I now drop the assumption that agent types are observable, and study the incentive-compatible efficient allocation from the constrained maximization problem, which I call SP^2,

$$\max_{c,L} \pi u(c_1) + (1-\pi)\Big((1-p)u(c_{2H}) + pu(c_{2L})\Big) \tag{4.3.9}$$

subject to

$$\pi c_1 \leq L \tag{4.3.10}$$

$$\pi c_1 + (1-\pi)c_{2H} = L + R_H(1-L) \tag{4.3.11}$$

$$\pi c_1 + (1-\pi)c_{2L} = L + R_L(1-L) \tag{4.3.12}$$

$$(1-p)u(c_{2H}) + pu(c_{2L}) \geq u(c_1) \tag{4.3.13}$$

where L denotes investment in the storage technology. The first constraint states that the withdrawal (consumption) by agents who claim to be type 1 has to be less or equal to the investment in storage technology. The next two constraints are the resource constraints for the two states of the world, $s \in \{H, L\}$, respectively, and they state that the total consumption by all agents shall not exceed the total return obtained from the two investment technologies. Because agent types are unobservable, late consumers can imitate early consumers, obtain consumption at date 1, and store it for consumption at date 2. The last constraint (the incentive compatibility (IC) constraint) ensures that type-2 agents do not want to misrepresent themselves as type-1 agents. Note that it is never optimal for type-1 agents imitate type-2 agents because type-1 agents do not value any consumption beyond date 1.

Lemma 4.1. When the utility function is exponential, $u(c) = -\frac{1}{\gamma}\exp(-\gamma c)$, the constrained efficient allocation as in SP^2 coincides with unconstrained efficient allocation as in SP^1. Therefore, a solution to SP^1 is also a solution to SP^2.

Similar results to Lemma 4.1 have been derived in other studies such as Farhi et al. (2009). The argument behind a proof of Lemma 4.1 is that without the IC constraint, the first order equations would lead to the IC constraint.

The first order condition with exponential utility function is given by

$$\exp(-\gamma c_1) = (1-p)R_H \exp\Big(-\gamma\frac{R_H(1-\pi c_1)}{1-\pi}\Big) + pR_L \exp\Big(-\gamma\frac{R_L(1-\pi c_1)}{1-\pi}\Big) \quad (4.3.14)$$

Let EU^{no} be the maximized utility with exponential utility form. This will be the "no interim information" benchmark for later comparison.

4.4 Loan Loss Provision

Now I introduce loan loss provision to the bank. I abstract from much institutional detail in order to develop a tractable model that captures the essence of loan loss provision: a liquidity reserve based on a signal about the quality of loans which induces an inter-temporal shift of resources available to depositors hit by different liquidity shocks.

Two features of loan loss provision are worth elaborating: (i) Loan loss provision reflects changes in bank management's estimate of the expected losses, where management's estimate is consistent with depositors' belief,

$$y(\hat{p}) \equiv E[\tilde{R}; p] - E[\tilde{R}; \hat{p}];$$

(ii) as a form of liquidity reserve, loan loss provision changes the structure of the demand-deposit contracts offered by the bank. Specifically, it reduces the resource available to early consumers by the amount of loan loss provision, and increases the resource available to late consumers by the same amount. Notice that such inter-temporal shift is *contractible* but *not deterministic*: "contractible" in the sense that demand-deposit contracts can be contingent on all possible values of provision, "not deterministic" in the sense that the level is dependent on a stochastic variable to be revealed at date 1. Therefore, at the time of making investment decisions, banks have limited information as to how much loan loss provision they will be required to make.

Another feature of the loan loss provision in my model that warrants explanation is that it is allowed to take negative values. A negative loan loss provision can be viewed as an improvement in the expectation of future payoff to the risky loan, or a reversal of previous provisions in the real world with multiple periods. Dugan (2009) notes that under the incurred loss model, negative loan loss provision could be resulted from inability of bank managers to provide convincingly bad historical records:

> "Unfortunately, using historical loss rates to justify significant provisions becomes more difficult in a prolonged period of benign economic conditions when loss rates decline. Indeed, the longer the benign period, the harder it is to use acceptable documentation based on history and recent experience to justify significant provisioning. When bankers were unable to produce such acceptable historical documentation, auditors began to lean on them either to reduce provisions, or, in some circumstances, to take the extreme step of reducing the loan loss reserve by releasing so-called 'negative provisions' that counted as earnings." - Dugan (2009)

In the following, I consider two cases: (i) loan loss provision based on perfect interim information; (ii) loan loss provision based on noisy information. Without loss of generality, I assume that $p = \frac{1}{2}$ in the following discussion.

4.4.1 Perfect interim information

I first consider an extreme case where loan loss provision is based on a perfect signal about loan payoff. At date 1, all agents and banks observe a signal $\hat{p} \in \{0, 1\}$, as an updated assessment of the probability of the low payoff, $p \equiv \Pr(s = L)$ (in other words, $s = H, L$ is observed). This signal fully resolves the uncertainty about loan payoff. Having observed this signal, in conjunction with their own realized types, agents decide whether to consume at date 1. Therefore, both type-1 and type-2 agents' consumption will be contingent on the interim signal. This information environment is similar to the benchmark setting of Allen and Gale (1998) and Chari and Jagannathan (1988), who assume that a perfectly accurate leading economic indicator is observed by all or a random fraction of type-2 agents at date 1. In other words, loan loss provision is based a perfect signal about the state, which can be denoted as y_s, where $s = H, L$,

$$y_H = (0 - \frac{1}{2})(R_H - R_L) = -\frac{R_H - R_L}{2} \tag{4.4.1}$$

$$y_L = (1 - \frac{1}{2})(R_H - R_L) = \frac{R_H - R_L}{2} \tag{4.4.2}$$

The introduction of loan loss provision reduces the available resource at date 1 and increases it at date 2, by the amount y_s.

Since the state of the world $s \in \{H, L\}$ is now perfectly known at date 1, all consumption levels have to be conditional on s. Therefore, the demand-deposit contract offered by the bank is assumed to take the form $c \equiv \left(c_{1H}, c_{1L}, c_{2H}, c_{2L}\right)$. The bank chooses deposit contracts c and makes investment decision L to maximizes the expected utility of a typical agent. The following problem, which I call SP^2_{full}, characterizes bank's constrained optimization problem with accurate loan loss provision,

$$\max_{c,L} \pi\Big((1-p)u(c_{1H}) + pu(c_{1L})\Big) + (1-\pi)\Big((1-p)u(c_{2H}) + pu(c_{2L})\Big) \tag{4.4.3}$$

subject to

$$\pi c_{1H} \le L - y_H \tag{4.4.4}$$

$$\pi c_{1L} \le L - y_L \tag{4.4.5}$$

$$\pi c_{1H} + (1-\pi)c_{2H} \le L + R_H(1-L) \tag{4.4.6}$$

$$\pi c_{1L} + (1-\pi)c_{2L} \le L + R_L(1-L) \tag{4.4.7}$$

$$c_{2H} \ge c_{1H} \tag{4.4.8}$$

$$c_{2L} \ge c_{1L} \tag{4.4.9}$$

where $y_H = -\frac{R_H - R_L}{2}$ and $y_L = \frac{R_H - R_L}{2}$.

Note that the resource constraints at both date 1 and date 2 have to be conditional on the realized state s, because s is already known at date 1. The incentive constraints state that regardless of the state of the world revealed at date 1, late consumers are always better off by truthfully reporting his type.

Lemma 4.2. When there is perfect interim information at date 1 about the uncertain payoff to the risky loan, the optimal demand-deposit contract offered by the bank, contingent on the investment in the storage technology (L) is $c_{1H} = c_{2H} = L + R_H(1 - L)$ and $c_{1L} = c_{2L} = L + R_L(1 - L)$.

Lemma 4.2 reduces the bank's optimization problem to a univariate maximization problem with respect to L,

$$\max_{0 \leq L \leq 1} (1 - p)u(L + R_H(1 - L)) + pu(L + R_L(1 - L)) \tag{4.4.10}$$

It is easy to show that I can obtain a closed-form solution for this full information case, as stated in the following proposition.

Proposition 4.1. When there is perfect interim information at date 1, the optimal investment in the storage technology is given by

$$\hat{L}^{\text{full}} = 1 - \frac{1}{\gamma(R_H - R_L)} \ln\left(\frac{(1 - p)(R_H - 1)}{p(1 - R_L)}\right) \tag{4.4.11}$$

and the resulted consumptions are $c_{1H} = c_{2H} = \hat{L}^{\text{full}} + R_H(1 - \hat{L}^{\text{full}})$ and $c_{1L} = c_{2L} = \hat{L}^{\text{full}} + R_L(1 - \hat{L}^{\text{full}})$, which will determine the maximum expected utility that can be achieved, which I denote as EU^{full}, where "full" is label for "full interim information".

4.4.2 Noisy interim information

The informational role of loan loss provision is tantamount to a noisy signal about p, while the regulatory role is tantamount to a liquidity reserve that changes the structure of the demand deposit contracts. At date 1 the bank makes a loan loss provision $y \in \mathbb{R}$, and discloses it to the depositors. After observing y, the depositors update their assessment of the payoff from p to $\hat{p} = \frac{1}{2} + \varepsilon$, where ε is uniformly distributed, $\varepsilon \sim U(-\frac{1}{2}, \frac{1}{2})$, with pdf $f(\varepsilon) = 1$ for $\varepsilon \in [-\frac{1}{2}, \frac{1}{2}]$. The loan loss provision can be described without loss of information by the posterior belief $\hat{p} \in [0, 1]$ to which it could lead. I assume that loan loss provision reflects changes in bank management's estimate of the expected losses, where management's estimate is consistent with depositors' belief,

$$y(\hat{p}) \equiv E[\tilde{R}; p] - E[\tilde{R}; \hat{p}] = (\hat{p} - \frac{1}{2})(R_H - R_L) = (R_H - R_L)\varepsilon \tag{4.4.12}$$

Therefore,

$$y \sim U\Big(-\frac{R_H - R_L}{2}, \frac{R_H - R_L}{2}\Big)$$

The assumption that the interim signal is noisy is rather radical as it appears, but it is not without empirical underpinning. Empirical studies have documented that loan loss provision is widely used as a tool to manage earnings or manage capital, and is therefore more opportunistic and random than it is intended.

Demand-deposit contract

The introduction of loan loss provision renders the demand-deposit contract contingent on the level of loan loss provision made at date 1. As Allen and Gale (1998) point out, a typical deposit contract is "noncontingent" in the sense that, ex ante no contingency is intended in the original contract, but ex post actual withdrawals may be contingent on whether the bank is still solvent. In this model, deposit contract exhibits both ex ante contingency (with respect to loan loss provision) and ex post contingency (with respect to solvency). Specifically, I assume that the demand-deposit contracts take the form

$$c(y) \equiv \Big(c_1(y), c_{2H}(y), c_{2L}(y)\Big)$$

where $c_1(y)$ is the date-1 consumption (withdrawal) of agents who claim to be type 1, given that the loan loss provision is y; and $c_{2s}(y)$ is the date-2 consumption of agents who claim to be type 2 given y, if state $s = H, L$ happens.

The introduction of loan loss provision reduces the available resource at date 1 and increases it at date 2, by the amount y,

$$\pi c_1(y) \le L - y \tag{4.4.13}$$

$$(1-\pi)c_{2H}(y) \le (L - y - \pi c_1(y)) + R_H(1-L) + y \tag{4.4.14}$$

$$(1-\pi)c_{2L}(y) \le (L - y - \pi c_1(y)) + R_L(1-L) + y \tag{4.4.15}$$

To ensure the deposit contracts never induce negative consumptions for any level of loan loss provision, I make the following assumption:

Assumption 4.1.

$$\frac{R_H - R_L}{2} < 1 - \pi \tag{4.4.16}$$

Bank's optimization problem SP^2_{LLP} (for simplicity, denote as P3) is

$$\max_{c(y),L} \int_{-\frac{R_H-R_L}{2}}^{\frac{R_H-R_L}{2}} \left(\pi u(c_1(y)) + (1-\pi)\Big((1-p)u(c_{2H}(y)) + pu(c_{2L}(y))\Big)\right)\frac{1}{R_H - R_L}dy \tag{4.4.17}$$

subject to

$$\pi c_1(y) \leq L - y \tag{4.4.18}$$

$$\pi c_1(y) + (1-\pi)c_{2H}(y) \leq L + R_H(1-L) \tag{4.4.19}$$

$$\pi c_1(y) + (1-\pi)c_{2L}(y) \leq L + R_L(1-L) \tag{4.4.20}$$

$$(1-p)u(c_{2H}(y)) + pu(c_{2L}(y)) \geq u(c_1(y)) \tag{4.4.21}$$

for $\forall y \in \big[-\frac{R_H-R_L}{2}, \frac{R_H-R_L}{2}\big]$.

Similar to the benchmark models, the incentive compatibility constraints in (P3) are dispensable. Let (P3') denote the "reduced form" problem without the IC constraints.

Lemma 4.3. When the utility function is exponential, (P3) is equivalent to (P3').

Assumption 4.2.

$$\frac{(1-\pi)\pi}{\gamma} \ln\Big(\exp(\frac{R_H - R_L}{1-\pi}) + 1\Big) - \pi R_L - \frac{R_H - R_L}{2} < 0 \tag{4.4.22}$$

Assumption 4.3.

$$0 < \frac{1}{\kappa}(R_H - R_L) - (R_H - 1) < \frac{1}{\pi} \tag{4.4.23}$$

where $\kappa = \exp\Big(-\gamma\frac{(R_H-R_L)(1-L)}{1-\pi}\Big) + 1$. Assumptions 4.2 and 4.3 hold for rather flexible parameter values.

Lemma 4.4. Suppose Assumptions 4.1-4.3 hold. When there is an uninformative loan loss provision at date 1, the optimal demand-deposit contract offered by the bank, contingent on the investment in the storage technology (L) is given by the following, Case (i): For $L \in [0, L^*]$,

$$c_1(y) = \begin{cases} L + R_L(1-L) - (1-\pi)\frac{\ln(\kappa)}{\gamma} & \text{for } y \in \left[-\frac{R_H - R_L}{2}, y^*\right) \\ \frac{L-y}{\pi} & \text{for } y \in \left[y^*, \frac{R_H - R_L}{2}\right] \end{cases}$$

$$c_{2H}(y) = \begin{cases} L + \frac{R_H - \pi R_L}{1-\pi}(1-L) + \frac{\pi \ln(\kappa)}{\gamma} & \text{for } y \in \left[-\frac{R_H - R_L}{2}, y^*\right) \\ \frac{R_H(1-L)+y}{1-\pi} & \text{for } y \in \left[y^*, \frac{R_H - R_L}{2}\right] \end{cases}$$

$$c_{2L}(y) = \begin{cases} L + R_L(1-L) + \pi\frac{\ln(\kappa)}{\gamma} & \text{for } y \in \left[-\frac{R_H - R_L}{2}, y^*\right) \\ \frac{R_L(1-L)+y}{1-\pi} & \text{for } y \in \left[y^*, \frac{R_H - R_L}{2}\right] \end{cases}$$

Case (ii): For $L \in (L^*, 1]$, $\forall y \in \left[-\frac{R_H - R_L}{2}, \frac{R_H - R_L}{2}\right]$,

$$c_1(y) = L + R_L(1-L) - (1-\pi)\frac{\ln(\kappa)}{\gamma} \tag{4.4.24}$$

$$c_{2H}(y) = L + \frac{R_H - \pi R_L}{1-\pi}(1-L) + \frac{\pi \ln(\kappa)}{\gamma} \tag{4.4.25}$$

$$c_{2L}(y) = L + R_L(1-L) + \pi\frac{\ln(\kappa)}{\gamma} \tag{4.4.26}$$

where $y^* = (1-\pi)\left(\frac{\pi \ln(\kappa)}{\gamma} + L\right) - \pi R_L(1-L)$, and L^* is implicitly defined by

$$(1-\pi)\left(\frac{\pi \ln(\kappa(L^*))}{\gamma} + L^*\right) - \pi R_L(1-L^*) = \frac{R_H - R_L}{2} \tag{4.4.27}$$

Figure 4.1 is a visual representation of Lemma 4.4 (for parameter values to be specified below), with four subfigures corresponding to four values of L: 0, 0.5, 0.9, and 1. The kinks in the first two subfigures happen at y^*, while the next two subfigures exhibit no kink because for these two values of L, $y^* > \frac{R_H - R_L}{2}$.

The following corollary explores how y^* (the "run threshold level") varies with the riskiness of the loan payoff.

Corollary 4.1. The threshold level y^* is inversely related to the variance of the payoff to the risky loan.

Proof.

$$\frac{dy^*}{d(R_H - R_L)} = \frac{\pi(1-\pi)}{\gamma\kappa}\frac{d\kappa}{d(R_H - R_L)} \tag{4.4.28}$$

but $\frac{d\kappa}{d(R_H-R_L)} = -\frac{\gamma(1-L)}{1-\pi}(\kappa - 1)$ and $\kappa > 1$, therefore,

$$\frac{dy^*}{d(R_H - R_L)} = -\pi(1-L)\frac{\kappa - 1}{\kappa} < 0 \tag{4.4.29}$$

Corollary 4.1 is similar to the main Proposition of Jacklin and Bhattacharya (1988), where (i) there is no role for loan loss provision and demand deposits are not contingent on loan loss provisions, and (ii) depositor preferences are smooth.

Risk sharing

Given the optimal demand-deposit contract as a function of L, as stated in Lemma 4.4, I can rewrite the expected utility as

$$EU(L) = \begin{cases} \int_{-\frac{R_H-R_L}{2}}^{y^*} u(c_1(y))\frac{1}{R_H-R_L}dy + \int_{y^*}^{\frac{R_H-R_L}{2}} \Psi(\cdot)\frac{1}{R_H-R_L}dy & \text{for } L \in [0, L^*] \\ \int_{-\frac{R_H-R_L}{2}}^{\frac{R_H-R_L}{2}} u(c_1(y))\frac{1}{R_H-R_L}dy & \text{for } L \in (L^*, 1] \end{cases}$$

where $\Psi(\cdot) = \pi u(c_1(y)) + (1-\pi)\big((1-p)u(c_{2H}(y)) + pu(c_{2L}(y))\big)$. The solution to the maximization problem

$$\max_{0 \le L \le 1} EU(L)$$

is given in the following proposition.

Proposition 4.2. Suppose Assumptions 4.1-4.3 hold. When there is an uninformative loan loss provision at date 1 and demand-deposit contracts are contingent on such signal, the optimal investment in the storage technology is 1.

This is a striking result because it is never optimal to invest any amount of resource in the risky loan. I can then obtain the consumption levels as functions of y, loan loss provision, $c_1(y)$, $c_{2H}(y)$, and $c_{2L}(y)$. This result contrasts with that of Allen and Gale (1998) where the interim signal is a perfect indicator of future asset returns.

I conduct a calibrated comparison of the case with loan loss provision with benchmark cases with different information structures. I calibrate the model as follows: $R_H = 1.5$, $R_L = 0.7$, $\pi = 0.5$, and $\gamma = 2$. In addition, as assumed throughout, $p = 0.5$.

(i) In the no information case, the optimal short-term investment is $\hat{L} = 0.5116$, the resulted consumption levels are $c_1 = 1.0232$, $c_{2H} = 1.4652$, $c_{2L} = 0.6838$. Expected utility is $EU^{\text{no}} = -0.0708$.

(ii) In the full information case, the optimal short-term investment is $\hat{L} = 0.6807$, the resulted consumption levels are $c_{1H} = c_{2H} = 1.1596$ and $c_{1L} = c_{2L} = 0.9042$, and the expected utility is $EU^{\text{full}} = -0.0656$.

(iii) In the noisy loan loss provision case, the optimal short-term investment is $\hat{L} = 1$, consumption levels are $c_1(y) = 0.8267$ and $c_{2H}(y) = c_{2L}(y) = 1.1733$, and the expected utility is $EU^{\text{LLP}} = -0.0957$.

Table 4.1 summarizes the results of the calibration exercise. Loan loss provision results in lower expected utility compared with alternative information structures:

$$EU^{\text{LLP}} < EU^{\text{no}} < EU^{\text{full}}$$

Robustness checks with alternative parameter values show that this ordering persists.

An plausible explanation for this result is in order. Loan loss provision essentially introduces contingency into the demand-deposit contract, and stipulates an inter-temporal shift of resource. Because of the uninformative nature of the signal based on which loan loss provision is made, such contingency distorts depositors' withdrawing decision and bank's investment decision. As I shall discuss in the following section, the damage on risk sharing could be greatest when loan loss provision is counter-cyclical.

4.4.3 The dark side of counter-cyclical provision

My finding that uninformative loan loss provision induces suboptimal risk sharing has broad implications for the regulatory debate on loan loss provision. Recall that the noisy interim signal is $\hat{p} = p + \varepsilon$, and loan loss provision reflects the revision in expected loan payoff conditional on this signal,

$$y(\hat{p}) \equiv E[\tilde{R}; p] - E[\tilde{R}; \hat{p}] = (R_H - R_L)\varepsilon \tag{4.4.30}$$

Definition 4.1. A loan loss provision is *ex post* counter-cyclical if it is conditional on realization of $\hat{p} = p + \varepsilon$ such that $\varepsilon \in [-p, 0)$ but the realized state is $s = L$, or $\varepsilon \in (0, 1-p]$ but the realized state is $s = H$.

A key feature of the counter-cyclical provision is that it is inversely related to the intended direction of loan loss provision if I were to provision for actual loan loss. In other words, more provision in good times and less provision in bad times. As I have shown, it is exactly the uninformativeness that leads to distortion in risk sharing and investment. If I follow regulators' call to make counter-cyclical provision, there might be unintended negative effects on risk sharing. Therefore, from a risk-sharing perspective, there is a justification for accountants' adherence to the informational role of loan loss provision and reluctance to make "counter-cyclical" loan loss provision.

4.5 Concluding Remarks

This paper incorporates loan loss provision in a variant of Diamond-Dybvig model, and solves for the optimal demand-deposit contract contingent on loan loss provision. I show that the existence of such contingency leads to underinvestment in risky loan and consequently lower expected utility for depositors.

There are at least several aspects of the model that can be generalized. In the current model I study two extreme cases of loan loss provision: entirely uninformative and perfectly informative loan loss provisions. An extension of the model could introduce variations in the level of informativeness of loan loss provision, and study its impact on risk sharing.

There are two conceivable limitations to the current setup. First, loan loss provision is conditioned on an exogenous, noisy interim signal, instead of being a choice variable of the bank manager. Future study could examine the optimal provisioning behavior of banks. Second, even though the model is rather tractable, the welfare comparison relies on calibrated numerical examples[4].

For regulatory purpose, loan loss provision is viewed as a form of tier 2 capital which technically influences whether a bank becomes insolvent. In the current setting, however, bank insolvency is circumvented because I focus on the incentive-compatible demand-deposit contracts which induce no runs. An interesting extension is to study the tradeoff between the two objectives of bank regulation: reducing insolvency risk and promoting risk sharing. This will greatly enrich the current regulatory policy deliberation which overly fixate on how to reduce procyclicality (and therefore, presumably, reduce insolvency risk).

[4]Gale and Özgür (2005) also resort to numerical examples.

4.6 Appendix

Proof of Proposition 4.1. The objective function is

$$EU(L) = -\frac{1}{\gamma}\Big((1-p)\exp\Big(-\gamma(L+R_H(1-L))\Big) + p\exp\Big(-\gamma(L+R_L(1-L))\Big)\Big) \tag{4.6.1}$$

The first order condition is

$$EU'(L) = -(1-p)(R_H-1)\exp\Big(-\gamma(L+R_H(1-L))\Big)+p(1-R_L)\exp\Big(-\gamma(L+R_L(1-L))\Big) = 0 \tag{4.6.2}$$

which is equivalent to

$$\frac{(1-p)(R_H-1)}{p(1-R_L)} = \exp\Big(\gamma(1-L)(R_H-R_L)\Big) \tag{4.6.3}$$

Therefore,

$$\hat{L}^{\text{full}} = 1 - \frac{1}{\gamma(R_H-R_L)}\ln\Big(\frac{(1-p)(R_H-1)}{p(1-R_L)}\Big) \tag{4.6.4}$$

It is easy to verify that the second order condition is satisfied.

Proof of Lemma 4.3. Let (P3') denote the optimization problem without the IC constraints (4.4.21), a necessary condition for a solution $(c(y), L)$ to (P3') is that, $\forall y$,

$$c(y) \in \text{argmax}_{c(y),L}\pi u(c_1(y)) + (1-\pi)\big((1-p)u(c_{2H}(y)) + pu(c_{2L}(y))\big) \tag{4.6.5}$$

subject to

$$\pi c_1(y) \le L - y \tag{4.6.6}$$

$$\pi c_1(y) + (1-\pi)c_{2H}(y) \leq L + R_H(1-L) \tag{4.6.7}$$

$$\pi c_1(y) + (1-\pi)c_{2L}(y) \leq L + R_L(1-L) \tag{4.6.8}$$

By applying the Kuhn-Tucker theorem, I form the Lagrangian function

$$\mathcal{L} = \pi u(c_1(y)) + (1-\pi)\big((1-p)u(c_{2H}(y)) + pu(c_{2L}(y))\big) + \lambda_1\big(L - y - \pi c_1(y)\big)$$

$$+\lambda_2\big(L+R_H(1-L) - \pi c_1(y) - (1-\pi)c_{2H}(y)\big) + \lambda_3\big(L+R_L(1-L) - \pi c_1(y) - (1-\pi)c_{2L}(y)\big)$$

First order conditions include equations (4.4.18)-(4.4.20), as well as

$$u'(c_1(y)) = \lambda_1 + \lambda_2 + \lambda_3 \tag{4.6.9}$$

$$(1-p)u'(c_{2H}(y)) = \lambda_2 \tag{4.6.10}$$

$$pu'(c_{2L}(y)) = \lambda_3 \tag{4.6.11}$$

$$\lambda_i \geq 0, \quad i = 1,2,3 \tag{4.6.12}$$

$$\lambda_1\big(L - y - \pi c_1(y)\big) = 0 \tag{4.6.13}$$

$$\lambda_2\big(L + R_H(1-L) - \pi c_1(y) - (1-\pi)c_{2H}(y)\big) = 0 \tag{4.6.14}$$

$$\lambda_3\big(L + R_L(1-L) - \pi c_1(y) - (1-\pi)c_{2L}(y)\big) = 0 \tag{4.6.15}$$

From (4.6.20)-(4.6.23),

$$\begin{aligned} u'(c_1(y)) &= \lambda_1 + (1-p)u'(c_{2H}(y)) + pu'(c_{2L}(y)) \\ &\geq (1-p)u'(c_{2H}(y)) + pu'(c_{2L}(y)) \end{aligned} \tag{4.6.16}$$

Given exponential utility function $u(c) = -\frac{1}{\gamma}\exp(-\gamma c)$, this is equivalent to

$$\exp(-\gamma c_1(y)) \geq (1-p)\exp(-\gamma c_{2H}(y)) + p\exp(-\gamma c_{2L}(y)) \tag{4.6.17}$$

By multiplying both sides by $-1/\gamma$, I have

$$-\frac{1}{\gamma}\exp(-\gamma c_1(y)) \leq (1-p)\Big(-\frac{1}{\gamma}\exp(-\gamma c_{2H}(y))\Big) + p\Big(-\frac{1}{\gamma}\exp(-\gamma c_{2L}(y))\Big) \tag{4.6.18}$$

which is the IC constraint (4.4.21).

Proof of Lemma 4.4.

Case (i): IC constraint holds with strict equality if $\pi c_1(y) < L - y$. Resource constraints (4.6.18)-(4.6.19) have to hold at equality, which implies that $c_{2H}(y) = \frac{L+R_H(1-L)-\pi c_1(y)}{1-\pi}$, $c_{2L}(y) = \frac{L+R_L(1-L)-\pi c_1(y)}{1-\pi}$; therefore $c_{2H}(y) = c_{2L}(y) + \frac{(R_H-R_L)(1-L)}{1-\pi}$.

Note that $(1-p)u(c_{2H}(y)) + pu(c_{2L}(y)) = u(c_1(y))$ implies that

$$\exp(-\gamma c_1(y)) = \kappa\exp(-\gamma c_{2L}(y))$$

where $\kappa \equiv \exp\Big(-\gamma\frac{(R_H-R_L)(1-L)}{1-\pi}\Big) + 1$. Therefore,

$$c_{2L}(y) = \frac{\ln(\kappa)}{\gamma} + c_1(y) \tag{4.6.19}$$

I restrict our search within the class of linear functions in loan loss provision, i.e., $c_1(y) = \alpha + \beta y$, where α and β are parameters to be pinned down. Therefore,

$$c_{2L}(y) = \frac{L + R_L(1-L) - \pi(\alpha + \beta y)}{1-\pi} \tag{4.6.20}$$

But by (4.6.30),

$$c_{2L}(y) = \frac{\ln(\kappa)}{\gamma} + (\alpha + \beta y) \tag{4.6.21}$$

Comparing the RHS of (4.6.31) and (4.6.32), I have

$$\begin{cases} \frac{L+R_L(1-L)-\pi\alpha}{1-\pi} - \frac{\ln(\kappa)}{\gamma} - \alpha = 0 \\ \frac{\pi\beta}{1-\pi} + \beta = 0 \end{cases}$$

Solving for α and β,

$$\begin{cases} \alpha = L + R_L(1-L) - (1-\pi)\frac{\ln(\kappa)}{\gamma} \\ \beta = 0 \end{cases}$$

Therefore,

$$c_1(y) = L + R_L(1-L) - (1-\pi)\frac{\ln(\kappa)}{\gamma} \tag{4.6.22}$$

$$c_{2L}(y) = L + R_L(1-L) + \pi\frac{\ln(\kappa)}{\gamma} \tag{4.6.23}$$

$$c_{2H}(y) = L + \frac{R_H - \pi R_L}{1-\pi}(1-L) + \frac{\pi \ln(\kappa)}{\gamma} \tag{4.6.24}$$

Let us now revisit the condition for Case (i). $\pi c_1(y) < L - y$ implies that

$$L + R_L(1-L) - (1-\pi)\frac{\ln(\kappa)}{\gamma} < \frac{L-y}{\pi} \tag{4.6.25}$$

which yields

$$y < (1-\pi)\Big(\frac{\pi \ln(\kappa)}{\gamma} + L\Big) - \pi R_L(1-L) \equiv y^*$$

Case (ii): IC constraint becomes strict inequality when $y \geq y^*$. In this case, $\lambda_1 > 0$, which implies that $c_1(y) = \frac{L-y}{\pi}$, and $c_{2H}(y) = \frac{R_H(1-L)+y}{1-\pi}$, $c_{2L}(y) = \frac{R_L(1-L)+y}{1-\pi}$.

So far I have implicitly assumed that y^* is always within the support of y, i.e.,

$$y^* \in \Big[-\frac{R_H - R_L}{2}, \frac{R_H - R_L}{2}\Big] \tag{4.6.26}$$

The following Lemma 4.5 states that this is not always true. Specifically, the threshold level for loan loss provision y^* is not always below the upper bound of the support of y, i.e., $\frac{R_H-R_L}{2}$. When $y^* > \frac{R_H-R_L}{2}$, only Case (i) discussed above is relevant. Therefore, in the search for the optimal L, I have to differentiate two cases where $y^* \in \Big[-\frac{R_H-R_L}{2}, \frac{R_H-R_L}{2}\Big]$ and $y^* > \frac{R_H-R_L}{2}$, respectively. The proof of Proposition 4.2 is based on this observation.

Lemma 4.5. Let L^* be the solution to $(1-\pi)\Big(\frac{\pi \ln(\kappa(L^*))}{\gamma} + L^*\Big) - \pi R_L(1-L^*) = \frac{R_H-R_L}{2}$. When $0 \leq L \leq L^*$,

$$y^* \in \Big[-\frac{R_H - R_L}{2}, \frac{R_H - R_L}{2}\Big]$$

When $L^* < L \leq 1$,

$$y^* > \frac{R_H - R_L}{2}$$

Proof of Lemma 4.5. First, observe that the threshold $y^*(L)$ is not always below the upper bound. When $L = 1$,

$$y^* = (1-\pi)\Big(\frac{\pi \ln 2}{\gamma} + 1\Big) > 1 - \pi \geq \frac{R_H - R_L}{2} \tag{4.6.27}$$

Define

$$f(L) = (1-\pi)\Big(\frac{\pi \ln(\kappa)}{\gamma} + L\Big) - \pi R_L(1-L) - \frac{R_H - R_L}{2}$$

By Assumption 4.2,

$$f(0) = (1-\pi)\frac{\pi \ln(\kappa)}{\gamma} - \pi R_L - \frac{R_H - R_L}{2} < 0 \tag{4.6.28}$$

By Assumption 4.1,

$$f(1) = (1-\pi) - \frac{R_H - R_L}{2} > 0 \tag{4.6.29}$$

By Assumption 4.3,

$$f'(L) = \pi\Big(R_H - 1 - \frac{1}{\kappa}(R_H - R_L)\Big) + 1 > 0 \quad \forall L \in [0,1] \tag{4.6.30}$$

Therefore, there must be a unique L^* such that $f(L^*) = 0$. Therefore, L^* is defined by

$$(1-\pi)\Big(\frac{\pi \ln(\kappa(L^*))}{\gamma} + L^*\Big) - \pi R_L(1-L^*) = \frac{R_H - R_L}{2} \tag{4.6.31}$$

Proof of Proposition 4.2. The expected utility can be expressed as

$$EU(L) = \begin{cases} \int_{-\frac{R_H-R_L}{2}}^{y^*} u(c_1(y))\frac{1}{R_H-R_L}dy + \int_{y^*}^{\frac{R_H-R_L}{2}} \Psi(\cdot)\frac{1}{R_H-R_L}dy & \text{for } 0 \leq L < L^* \\ \int_{-\frac{R_H-R_L}{2}}^{\frac{R_H-R_L}{2}} u(c_1(y))\frac{1}{R_H-R_L}dy & \text{for } L^* \leq L \leq 1 \end{cases}$$

where $\Psi(\cdot) = \pi u(c_1(y)) + (1-\pi)\big((1-p)u(c_{2H}(y)) + pu(c_{2L}(y))\big)$.

To find the solution to the maximization problem

$$\max_{0 \leq L \leq 1} EU(L)$$

I look at the two cases.

Case (i): When $0 \leq L \leq L^*$, the expected utility function is

$$
\begin{aligned}
EU(L) =& \int_{-\frac{R_H-R_L}{2}}^{y^*} u(c_1(y))\frac{1}{R_H-R_L}dy + \int_{y^*}^{\frac{R_H-R_L}{2}} \Big(\pi u(c_1(y)) + (1-\pi)((1-p)u(c_{2H}(y)) + pu(c_{2L}(y)))\Big)\frac{1}{R_H-R_L}dy \\
=& \frac{1}{R_H-R_L}\Big[\int_{-\frac{R_H-R_L}{2}}^{y^*} \Big(-\frac{1}{\gamma}\exp(-\gamma[L+R_L(1-L)-(1-\pi)\frac{\ln(\kappa)}{\gamma}])\Big)dy \\
&+ \int_{y^*}^{\frac{R_H-R_L}{2}} \Big(\pi\Big(-\frac{1}{\gamma}\exp(-\gamma\frac{L-y}{\pi})\Big) - (1-\pi)\Big(\frac{1-p}{\gamma}\exp(-\gamma\frac{R_H(1-L)+y}{1-\pi}) + \frac{p}{\gamma}\exp(-\gamma\frac{R_L(1-L)+y}{1-\pi})\Big)\Big)dy\Big] \\
=& \frac{1}{R_H-R_L}\Big[\Big(-\frac{1}{\gamma}\exp(-\gamma[L+R_L(1-L)-(1-\pi)\frac{\ln(\kappa)}{\gamma}])\Big)(y^*+\frac{R_H-R_L}{2}) + \frac{\pi^2}{\gamma^2}\exp(-\gamma\frac{L-y}{\pi})\Big|_{y^*}^{\frac{R_H-R_L}{2}} \\
&+ \frac{(1-\pi)^2(1-p)}{\gamma^2}\exp(-\gamma\frac{R_H(1-L)+y}{1-\pi})\Big|_{y^*}^{\frac{R_H-R_L}{2}} + \frac{(1-\pi)^2 p}{\gamma^2}\exp(-\gamma\frac{R_L(1-L)+y}{1-\pi})\Big|_{y^*}^{\frac{R_H-R_L}{2}}\Big] \\
=& \frac{1}{R_H-R_L}\Big[\Big(-\frac{1}{\gamma}\exp(-\gamma[L+R_L(1-L)-(1-\pi)\frac{\ln(\kappa)}{\gamma}])\Big)(y^*+\frac{R_H-R_L}{2}) \\
&+ \frac{\pi^2}{\gamma^2}\Big(\exp(-\gamma\frac{L-(R_H-R_L)/2}{\pi}) - \exp(-\gamma\frac{L-y^*}{\pi})\Big) \\
&+ \frac{(1-\pi)^2}{\gamma^2}\Big((1-p)\Big(\exp(-\gamma\frac{R_H(1-L)+(R_H-R_L)/2}{1-\pi}) - \exp(-\gamma\frac{R_H(1-L)+y^*}{1-\pi})\Big) \\
&+ p\Big(\exp(-\gamma\frac{R_L(1-L)+(R_H-R_L)/2}{1-\pi}) - \exp(-\gamma\frac{R_L(1-L)+y^*}{1-\pi})\Big)\Big)\Big]
\end{aligned}
\tag{4.6.32}
$$

It can be shown that this function is increasing in L. Therefore, the maximum expected utility is achieved at $L = L^*$ for $0 \leq L \leq L^*$. When $L = L^*$, $y^* = \frac{R_H-R_L}{2}$, and

$$EU(L^*) = u(c_1(L^*)) \tag{4.6.33}$$

Case (ii): When $L^* < L \leq 1$, again, the maximization problem becomes equivalent to

$$\max_{L^* \leq L \leq 1} c_1(L) = L + R_L(1-L) - (1-\pi)\frac{\ln(\kappa)}{\gamma} \tag{4.6.34}$$

By Assumption 4.3,

$$c_1'(L) = \frac{1}{\kappa}(R_H - R_L) - (R_H - 1) > 0$$

So the maximum expected utility is achieved at $L = 1$ for $L^* < L \leq 1$. Combining both cases, $\hat{L} = 1$ is the global maxim.

4.7 References

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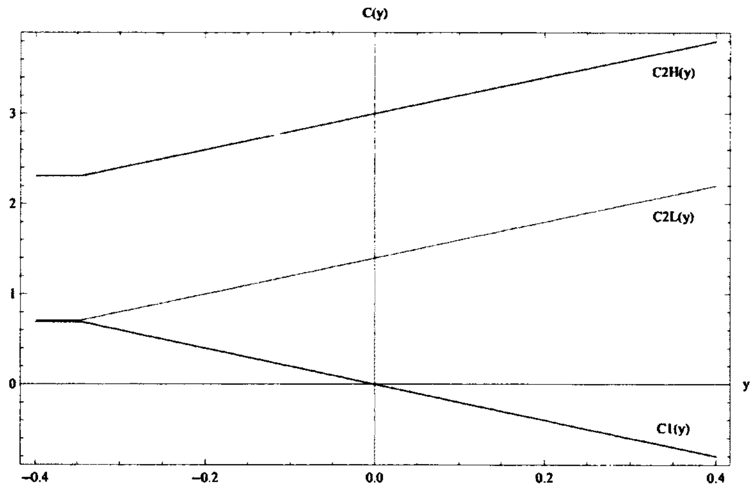
Table 4.1: Calibrated Comparison

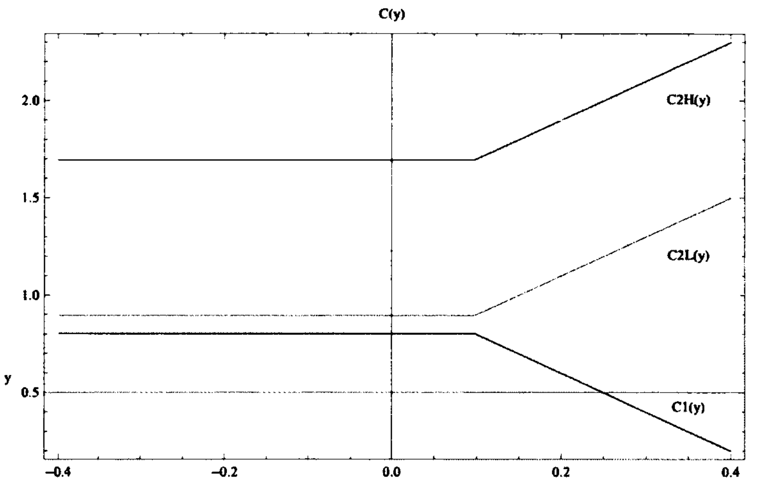
This table reports calibrated comparison of different information structures using the following parameter values: $R_H = 1.5$, $R_L = 0.7$, $p = 0.5$, $\gamma = 2$, $\pi = 0.5$.

Information environment	No information	Full information	Loan loss provision
Deposit contracts	(c_1, c_{2H}, c_{2L})	$(c_{1H}, c_{1L}, c_{2H}, c_{2L})$	$(c_1(y), c_{2H}(y), c_{2L}(y))$
Optimal investment in storage	$\hat{L} = 0.5116$	$\hat{L} = 0.6807$	$\hat{L} = 1$
Resulted consumption	$c_1 = 1.0232$, $c_{2H} = 1.4652$, $c_{2L} = 0.6838$	$c_{1H} = c_{2H} = 1.1596$, $c_{1L} = c_{2L} = 0.9042$	$c_1(y), c_{2H}(y), c_{2L}(y)$ as in Lemma 6
Expected utility	$EU^{\text{no}} = -0.0708$	$EU^{\text{full}} = -0.0656$	$EU^{\text{LLP}} = -0.0957$

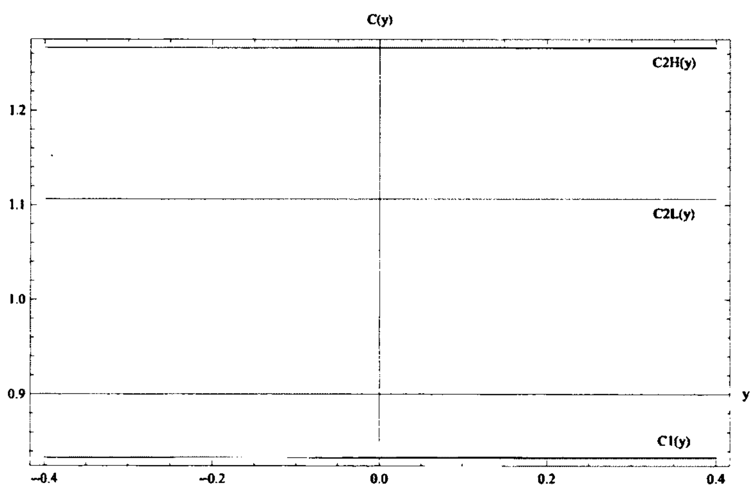
Figure 4.1: Demand-deposit Contract Contingent on Loan Loss Provision



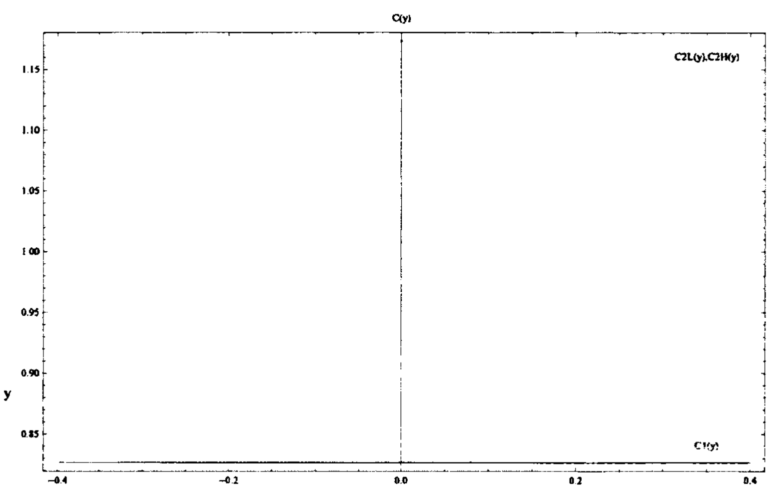
(a) $L = 0$



(b) $L = 0.5$



(c) $L = 0.9$



(d) $L = 1$

Chapter 5

Concluding Remarks

This dissertation examines three new settings of earnings management and public disclosure. By solving an infinite-horizon reporting problem, the first essay shows that different investor expectations schemes have important implications for the dynamics of earnings management and asset prices. The second essay studies the role of public disclosure in a coordination game with trading, and illustrates the intricate role of public disclosure on equilibrium determinacy and welfare. The third essay studies how the introduction of loan loss provision affects the risk-sharing function of a bank.

Even though the three essays provide empirical predictions or policy implications, cautions need to be taken when interpreting the results. For example, the first essay is based on a simple binary-earnings model, and may not be suitable to be calibrated with corporate earnings data in the real world. In fact, most of the findings in my dissertation are intended to provide qualitative rather than quantitative insights.

This dissertation is not intended to be a complete or comprehensive study of the three settings identified above. There are other aspects of the settings that are beyond the scope of my research. Specifically, the first essay does not allow different investor expectations to coexist, and does not address the mechanism design problem which explores the optimal compensation contract. The second essay focuses on the information aggregation role of the secondary market, but does not explicitly address the empty creditor problem. The third essay studies the optimal demand-deposit contract but does not explicitly model the probability of bank run.

There are also related questions which are on the agenda of my future research. Related to the second essay, I would like to investigate the empirical relationship between the disclosure quality of distress firms and their Chapter 11 outcomes. Related to the third essay, I intend to study the impact of accounting measurement rules concerning loan loss provision on banks' risk sharing.[1]

The three essays make the first systematic attempt to address accounting questions from perspectives of macroeconomics. In spite of the fact that the relation between accounting and macroeconomics is as old as the existence of national accounting, the two academic professions have largely evolved in isolation. However, I believe that modern macroeconomics offers techniques and perspectives which could extend the scope of accounting research. My dissertation is a testament to this belief. For instances, the first essay applies dynamic programming methods to a financial reporting problem; the second essay studies commitment to public disclosure precision in a global games setting.

[1]This is the focus of an ongoing research project with Haresh Sapra.